Time, measurement and information loss in quantum cosmology

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Abstract

A framework for a physical interpretation of quantum cosmology appropriate to a nonperturbative hamiltonian formulation is proposed. It is based on the use of matter fields to define a physical reference frame. In the case of the loop representation it is convenient to use a spatial reference frame that picks out the faces of a fixed simplicial complex and a clock built with a free scalar field. Using these fields a procedure is proposed for constructing physical states and operators in which the problem of constructing physical operators reduces to that of integrating ordinary differential equations within the algebra of spatially diffeomorphism invariant operators. One consequence is that we may conclude that the spectra of operators that measure the areas of physical surfaces are discrete independently of the matter couplings or dynamics of the gravitational field.

Using the physical observables and the physical inner product, it becomes possible to describe singularities, black holes and loss of information in a nonperturbative formulation of quantum gravity, without making reference to a background metric. While only a dynamical calculation can answer the question of whether quantum effects eliminate singularities, it is conjectured that, if they do not, loss of information is a likely result because the physical operator algebra that corresponds to measurements made at late times must be incomplete.

Finally, I show that it is possible to apply Bohr's original operational interpretation of quantum mechanics to quantum cosmology, so that one is free to use either a Copenhagen interpretation or a corresponding relative state interpretation in a canonical formulation of quantum cosmology.

Contents

1. Introduction

What happens to the information contained in a star that collapses to a black hole, after that black hole has evaporated? This question, perhaps more than any other, holds the key to the problem of quantum gravity. Certainly, no theory could be called a successful unification of quantum theory and general relativity that does not confront it. Nor does it seem likely that this can be done without the introduction of new ideas. Furthermore, in spite of the progress that has been made on quantum gravity on several fronts over the last years, and in spite of some recent attention focused directly on it[1] this problem remains at this moment open.

In this paper I would like to ask how this problem may be addressed from the point of view of one approach to quantum gravity, which is the nonperturbative approach based on canonical quantization[AA91, CR91, LS91]. This approach has been under rapid development for the last several years in the hopes of developing a theory that could address such questions from first principles. What I hope to show here is that this approach has recently come closer to being able to address problems of physics and cosmology. To illustrate this, I hope to show here that the canonical approach may lead to new perspectives about the problem of what happens when a black hole evaporates that come from thinking carefully about how such questions can be asked from a purely diffeomorphic and nonperturbative point of view.

One reason why the canonical approach has not, so far, had much to say about this and other problems is that there is a kind of discipline that comes from working completely within a nonperturbative framework that, unfortunately, tends to damp certain kinds of intuitive or speculative thinking about physical problems. This is that, as there is no background geometry to make reference to, one cannot say anything about physics unless it is said using physical operators, states and inner products. Unfortunately, while we have gained some nontrivial information about the physical states of the theory, there has been, until recently, rather little progress about the problem of constructing physical operators.

From a conceptual point of view, the problem of the physical observables is difficult because it is closely connected to the problem of time[2]. It is difficult to construct physical observables because in a diffeomorphism invariant theory one cannot be naive about where and when an observation takes place. Coordinates have no meaning so that, to be physically meaningful, an operator must locate the information it is to measure by reference to the physical configuration of the system. Of course, this is not necessary if we are interested only in global, or topological, information about the fields, but as general relativity is a local field theory, with local degrees of freedom, and as we are local observers, we must have a practical way to construct operators that describe local measurements if we are to have a useful quantum theory of gravity.

Thus, to return to the opening question, if we are, within a nonperturbative framework, to ask what happens *after a black hole evaporates,* we must be able to construct spacetime

[1] An unsystematic sampling of the interesting papers that have recently appeared are [CGHS, EW91, B92, GH92, DB92, HS92] [JP92, SWH92, ST92, 'tH85].

[2] Two good recent reviews of the problem of time with many original references are [KK92, I92]. The point of view pursued here follows closely that of Rovelli in [CRT91].

diffeomorphism invariant operators that can give physical meaning to the notion of *after the evaporation.* Perhaps I can put it in the following way: the questions about loss of information or breakdown of unitary evolution rely, implicitly, on a notion of time. Without reference to time it is impossible to say that something is being lost. In a quantum theory of gravity, time is a problematic concept which makes it difficult to even ask such questions at the nonperturbative level, without reference to a fixed spacetime manifold. The main idea, which it is the purpose of this paper to develop, is that the problem of time in the nonperturbative framework is more than an obstacle that blocks any easy approach to the problem of loss of information in black hole evaporation. It may be the key to its solution.

As many people have argued, the problem of time is indeed the conceptual core of the problem of quantum gravity. Time, as it is conceived in quantum mechanics is a rather different thing than it is from the point of view of general relativity. The problem of quantum gravity, especially when put in the cosmological context, requires for its solution that some single concept of time be invented that is compatible with both diffeomorphism invariance and the principle of superposition. However, looking beyond this, what is at stake in quantum gravity is indeed no less and no more than the entire and ancient mystery: *What is time?* For the theory that will emerge from the search for quantum gravity is likely to be the background for future discussions about the nature of time, as Newtonian physics has loomed over any discussion about time from the seventeenth century to the present.

I certainly do not know the solution to the problem of time. Elsewhere I have speculated about the direction in which we might search for its ultimate resolution[LS92c]. In this paper I will take a rather different point of view, which is based on a retreat to what both Einstein and Bohr taught us to do when the meaning of a physical concept becomes confused: reach for an operational definition. Thus, in this paper I will adopt the point of w that time is precisely no more and no less than that which is measured by physical clocks. From this point of view, if we want to understand what time is in quantum gravity then we must construct a description of a physical clock living inside a relativistic quantum mechanical universe.

This is, of course, an old idea. The idea that physically meaningful observables in general relativity may be constructed by introducing a physical reference system was introduced by Einstein[AE]. To my knowledge it was introduced to the literature on quantum gravity in a classic paper of DeWitt[BD62] and has recently been advocated by Rovelli[CRM91], Kuchar and Torre[KT91], Carlip[SC92] and other authors. However, what I hope becomes clear from the following sections is that this is not just a nice idea which can be illustrated in simple model systems with a few degrees of freedom. There is, I believe, a good chance that this proposal can become the heart of a viable strategy to construct physical observables, states and inner products in the real animal-the quantum theory of general relativity coupled to an arbitrary set of matter fields. Whether any of those theories really exist as good diffeomorphism invariant quantum field theories is, of course, not settled by the construction of an approach to their interpretation. However, what I think emerges from the following is a workable strategy to construct the theory in a way that, if the construction works, what we will have in our hands is a physical theory with a clear interpretation.

The interpretational framework that I will be proposing is based on both technical and conceptual developments. On the technical side, I will be making use of recent developments that allow us to construct finite operators that represent diffeomorphism invariant quantitites[CR93, LS93, VH93]. These include spatially diffeomorphism invariant operators that measure geometrical quantities such as the areas of surfaces picked out by the configurations of certain matter fields. By putting this together with a simple physical model of a field of synchronized clocks, we will see that we are able to implement in full quantum general relativity the program of constructing physical observables based on quantum reference systems .

It may be objected that real clocks and rulers are much more complicated things than those that are modeled here; in reality they consist of multitudes of atoms held together by electromagnetic interactions. However, my goal here is precisely to show that useful results can be achieved by taking a shortcut in which the clocks and rulers are idealized and their dynamics simplified to the point that their inclusion into the nonperturbative dynamics is almost trivial. At the same time, no simplifications or approximations of any kind are made concerning the dynamics of the gravitational degrees of freedom. What we will then study is a system in which toy clocks and rulers interact with the fully nonlinear gravitational field within a nonperturbative framework.

However, while I will be using toy clocks and rulers, the main results will apply equally to any system in which certain degrees of freedom can be used to locate events relative to a physical reference system. The chief of these results is that the construction of physical observables need not be the very difficult problem that it has sometimes been made out to be. In particular, it is not necessary to exactly integrate Einstein's equations to find the observables of the coupled gravity-reference matter system. Instead, I propose here an alternative approach which consists of the following steps: i) Construct a large enough set of spatially diffeomorphism invariant operators to represent any observations made with the help of a spatial physical reference system; ii) Find the reality conditions among these spatially diffeomorphism invariant operators, and find the diffeomorphism invariant inner product that implements them; iii) Construct the projection of the Hamiltonian constraint as a finite and diffeomorphism invariant operator on this space. iv) Add degrees of freedom to correspond to a clock, or to a field of synchronized clocks. The Hamiltonian constraint for both states and operators now become ordinary differential equations for one parameter families of states and operators in the diffeomorphism invariant Hilbert space parametrized by the physical time measured by this clock. v) Define the physical inner product from the diffeomorphism invariant inner product by identifying the physical inner products of states with the diffeomorphism invariant inner products of their data at an initial physical time.

The steps necessary to implement this program are challenging. But the recent progress, concerning both spatially diffeomorphism invariant operators [CR93, LS93, VH93] and the form of the Hamiltonian constraint operator[RS88, BP92, RG90, Bl] suggests to me that each step can be accomplished. If so, then we will have a systematic way to construct the physical theory, together with a physical interpretation, from the spatially diffeomorphism invariant states and operators.

In the next section I review recent results about spatially diffeomorphism invariant observables which allow us to implement the idea of a spatial frame of reference. In section 3

I show how a simple model of a field of clocks can be used to promote these to physical observables[3].

Let me then turn from technical developments to conceptual developments. As is well known, there are two kinds of spatial boundary conditions that may be imposed in a canonical approach to quantum gravity: the open and the closed, or cosmological. The use of open boundary conditions, such as asymptotic flatness, avoids some of the main conceptual issues of quantum gravity because there is a real Hamiltonian which is tied to the clock of an observer outside the system, at spatial infinity. However, the asymptotically flat case also introduces additional difficulties into the canonical quantization program, so that it has not, so far, really helped with the construction of the full theory[4]. Furthermore, it can be argued that the asymptotically flat case represents an idealization that, by breaking the diffeomorphism invariance and postulating a classical observer at infinity, avoids exactly those problems which are the keys to quantum gravity. Thus, for both practical and philosophical reasons, it is of interest to see if it is possible to give a physical interpretation to quantum gravity in the cosmological context.

There has been a great deal of discussion recently about the interpretational problems of quantum cosmology[5]. However, most of it is not directly applicable to the project of this paper, either because it is tied to the path integral approach to quantum gravity, because it is applicable only in the semiclassical limit or because it breaks, either explicitly or implicitly, with the postulate that only operators that commute with the Hamiltonian constraint can correspond to observable quantities. What is required to turn canonical quantum cosmology into a physical theory is an interpretation in terms of expectation values, states and operators that describes what observers inside the universe can measure.

At the time I began thinking about this problem it seemed to me likely that what was required was some modification of the relative state idea of Everett[HE57], perhaps along the lines sketched in [LS84], which avoided commitment to the metaphysical idea of "many worlds" and incorporated some of the recent advances in understanding of the phenomena of "decoherence"[HPMZ]. The reason for this was that it seemed that the original interpretation of quantum mechanics, as developed by Bohr, Heisenberg, von Neumann and others could not be applied in the cosmological context. However, I have come to believe that this is too hasty a conclusion, and that, at least in the context in which physical observables are constructed by explicit reference to a physical reference frame and physical clocks, it is possible to apply directly to quantum cosmology the point of view of the original founders of quantum mechanics. The key idea is, as Bohr always stressed [B], to keep throughout the discussion an entirely operational point of view, so that the quantum state is never taken as a description of physical reality but is, instead, part of a description of a process of preparation and measurement involving a whole, entangled system including both the quantum system and the measuring devices.

[3]In an earlier draft of this paper there was an error in the treatment of the gauge fixed quantization in this section. The present treatment corrects the error and is, in addition, considerably simplified with respect to the original version.

[4]However, there are some interesting developments along this line, see [JB92].

[5]See, for example, [GMH, HPMZ, AS91].

I want to make it clear from the start that I do not intend here to take up the argument about different interpretations of quantum mechanics. In either ordinary quantum mechanics or in quantum cosmology there may be good reasons to prefer another interpretation over the original interpretation of Bohr. What I want to argue here is only that the claim that it is necessary to give up Bohr and von Neumann's interpretation in order to do quantum cosmology is wrong. As in ordinary quantum mechanics, once a strictly operational interpretation such as that of Bohr and von Neumann has been established, one can replace it with any other interpretation that makes more substantive claims about physical reality, whether it be a relative state interpretation, a statistical interpretation, or anything else. For this reason, I will give, in section 4, a sketch of an interpretation of quantum cosmology following the original language of Bohr and von Neumann. The reader who wants to augment this with the more substantive language of Everett, or of decoherence, will find that they can do so, in quantum cosmology no more and no less than in ordinary quantum mechanics[6].

Of course, one test that any proposed interpretation of quantum cosmology must satisfy is that it give rise to conventional quantum mechanics and quantum field theory in the appropriate limits. In section 5 I show how ordinary quantum field theory can be recovered by taking limits in which the gravitational degrees of freedom are treated semiclassically.

Having thus set out both the technical foundations and the conceptual bases of a physical interpretation of quantum cosmology, we will then be in a position to see what a fully nonperturbative approach may be able to contribute to the problems raised by the existence of singularities and the evaporation of black holes[SWH75]. While I will certainly not be able to resolve these problems here, it is possible to make a few preliminary steps that may clarify how these problems may be treated within a nonperturbative quantization. In particular, it is useful to see whether there are ways in which the existence of singularities and loss of information or breakdown of quantum coherence could manifest themselves in a fully nonperturbative treatment that does not make reference to any classical metric.

What I will show in section 6 is that there are useful notions of singularity and loss of information that make sense at the nonperturbative level. As there is no background metric, these must be described completely in terms of certain properties of the physical operator algebra. The main result of this section is that this can be done within the context of the physical reference systems developed in earlier sections. Furthermore, one can see at this level a relationship between the two phenomena, so that it seems likely the existence of certain kinds of singularities in the physical operator algebra can lead to effects that are naturally described as "loss of information." These results indicate that the occurrence of singularities and of loss of information are not necessarily inconsistent with the principles of quantum mechanics and general relativity. Whether they actually occur is then a dynamical question; it is possible that some consistent quantum theories of gravity allow the existence of singularities and the resulting loss of information, while others do not.

[6] The question of modeling the measurement process in parametrized systems is discussed in a paper in this volume by Anderson[AAn93]. Although Anderson warns against a too naive application of the projection postulate that does not take into account the fact that measurements take a finite amount of time, I do not think there is any inconsistency between his results and those of the present paper.

In order to focus the discussion, the results of section 6 are organized by the statements of two conjectures, which I call the *quantum singularity conjecture* and the *quantum cosmic censorship conjecture*. They embody the conditions under which we would want to say that the full quantum theory of gravity has singularities and the consequent loss of information.

The concluding section of this paper then focuses on two questions. First, are there approaches to an interpretation of quantum cosmology which, not being based on an operational notion of time, may avoid some of the limitations of the interpretation proposed here? Second, are there models and reductions of quantum cosmology in which the ideas presented here may be tested in detail?

It is an honor to contribute this paper to a volume in honor of Dieter Brill, who I have known for 16 years, first as a teacher of friends, then as a colleague as I became a frequent visitor to the Maryland relativity group. I am grateful for the warm hospitality I have felt from Dieter and the Maryland group on my many visits there.

2. A quantum reference system

In this section I will describe one example of a quantum reference system in which relative spatial positions are fixed using the configurations of certain matter fields. While I mean for this example to serve as a general paradigm for how reference systems might be described in quantum cosmology, I will use a coupling to matter and a set of observables that we have recently learned can be implemented in nonperturbative quantum gravity. Although I do not give the details here, every operator described in this section may be constructed by means of a regularization procedure, and in each case the result is a finite and diffeomorphism invariant operator[CR93, LS93, VH93].

In this section I will speak informally about preparations and measurements, however the precise statements of the measurement theory are postponed to section 4; as this depends on the operational notion of time introduced in the next section.

In this and the following sections, I am describing a canonical quantization of general relativity coupled to a set of matter fields. The spatial manifold, Σ, has fixed topological and differential structure, and will be assumed to be compact. For definiteness I will make use of the loop representation formulation of canonical quantum gravity[AA86, RS88]. Introductions to that formalism are found in [AA91, CR91, LS91]; summaries of results through the fall of 1992 are found in [AA92, LS92b].

2.1. Some operators invariant under spatial diffeomorphisms

In the last year we have found that while it seems impossible to construct operators that measure the gravitational field at a point, there exist well defined operators that measure nonlocal observables such as areas, volumes and parallel transports. In order to make these invariant under spatial diffeomorphisms we can introduce a set of matter fields which will label sets of open surfaces in the three manifold Σ. I will not here give details of how this is done, but ask the reader to assume the existence of matter fields whose configurations can be used to label a set of N open surfaces, which I will call $\mathcal{S}_I$, where $I = 1, ..., N$. The boundaries to these surfaces will also play a role, these are denoted $\partial\mathcal{S}_I$.

There are actually three ways in which such surfaces can be labeled by matter fields. One can use scalar fields, as described by Rovelli in [CR93] and Husain in [VH93], one can use antisymmetric tensor gauge fields, as is described in [LS93] or one can use abelian gauge fields in the electric field representation, as discussed by Ashtekar and Isham [AI92]. In each case we can construct finite diffeomorphism invariant operators which measure either the areas of these surfaces or the parallel transport of the spacetime connection around their boundaries. I refer the reader to the original papers for the technical details.

The key technical point is that, in each of these cases, the matter field can be quantized in a *surface representation*, in which the states are functionals of a set of N open surfaces in the three dimensional spatial manifold Σ[RG86, LS93]. For each of the N matter fields, a general bra will then be labeled by an unordered open surface, which may be disconnected, and will be denoted $< S_I|$ We assume that the states in the surface representation satisfy an identity which is analogous to the Abelian loop identities [GT81, AR91, AI92]. This is that whenever two, possibly disconnected, open surfaces S^1 and S^2 satisfy, for every two form F_{ab}, $\int_{S^1} F = \int_{S^2} F$, we require that $< S_I^1| =< S_I^2|$.

A general bra for all N matter fields is then labeled by N such surfaces, and will be denoted $< \mathcal{S}| =< \mathcal{S}_1, ..., \mathcal{S}_N|$ so that the general state may be written

$$\Psi[\mathcal{S}] =< \mathcal{S}|\Psi > \tag{1}$$

It is easy to couple this system to general relativity using the loop representation[RS88]. The gravitational degrees of freedom are incorporated by labeling the states by both surfaces and loops, so that a general state is of the form

$$\Psi[\gamma, \mathcal{S}] =< \gamma, \mathcal{S}|\Psi > \tag{2}$$

where γ is a loop in the spatial manifold Σ. We assume that all of the usual identities of the loop representation [RS88, AA91, CR91, LS91] are satisfied by these states.

The next step is to impose the constraints for spatial diffeomorphism invariance. By following the same steps as in the pure gravity case, it is easy to see that the exact solution to the diffeomorphism constraints for the coupled matter-gravity system is that the states must be functions of the diffeomorphism equivalence classes of loops and N labeled (and possibly disconnected) surfaces. As in the case of pure gravity the set of these equivalence classes is countable. If we denote by $\{\gamma, \mathcal{S}\}$ these diffeomorphism equivalence classes, every diffeomorphism invariant state may be written,

$$\Psi[\{\gamma, \mathcal{S}\}] =< \{\gamma, \mathcal{S}\}|\Psi > \tag{3}$$

Later we will include other matter fields, which will be denoted generically by ϕ. In that case states will be labeled by diffeomorphism equivalence classes, which I will denote by $\{\mathcal{S}, \gamma, \phi\}$. The space of such states will be denoted $\mathcal{H}_{diffeo}$.

A word of caution must be said about the notation: the expression $< \{\gamma, \mathcal{S}\}|\Psi >$ is not to be taken as an expression of the inner product. Instead it is just an expression for the action of the bras states on the kets. The space of kets is taken to be the space of functions $\Psi[\{\mathcal{S},\gamma\}]$ of diffeomorphism invariant classes of loops and surfaces. The space of bras are defined to be linear maps from this space to the complex numbers, and given some bra $<\chi|$, the map is defined by the pairing $<\chi|\Psi>$. The space of bras has thus a natural basis which is given by the $<\{\gamma,\mathcal{S}\}|$, whose action is defined by (3). For the moment, the inner product remains unspecified, so there is no isomorphism between the space of bras and kets. At the end of this section I will give a partial specification of the inner product.

On this space of states it is possible to construct two sets of diffeomorphism invariant observables to measure the gravitational field. The first of these are the areas of the I surfaces, which I will denote $\hat{\mathcal{A}}^I$. Operators which measure these areas can be constructed in the loop representation. The details are given in [CR93, LS93] where it is shown that after an appropriate regularization procedure the resulting quantum operators are diffeomorphism invariant and finite.

The bras $<\{\gamma,\mathcal{S}\}|$ are in fact eigenstates of the area operators, as long as the loops do not have intersections exactly at the surfaces. In references [CR93, LS93] it is shown that in this case,

$$<\{\gamma,\mathcal{S}\}|\hat{\mathcal{A}}^I = \frac{l^2_{Planck}}{2}\mathcal{I}^+[\mathcal{S}^I,\gamma] <\{\gamma,\mathcal{S}\}| \tag{4}$$

where $\mathcal{I}^+[\mathcal{S}^I,\gamma]$ is the unoriented, positive, intersection number between the surface and the loop that simply counts the intersections between them[7].

A second diffeomorphism invariant observable that can be constructed is the Wilson loop around the boundary of the I'th surface, which I will denote $\hat{T}^I$. As shown in [LS93] it has the action

$$<\{\gamma,\mathcal{S}\}|\hat{T}^I =<\{\gamma\cup\partial\mathcal{S}^I,\mathcal{S}\}| \tag{5}$$

In addition, diffeomorphism invariant analogues of the higher loop operators have recently been constructed by Husain [VH93].

It is easy to see that the algebra of these operators has the form,

$$[\hat{T}^J,\hat{\mathcal{A}}^I] = \frac{l^2_{Planck}}{2}\mathcal{I}^+[\mathcal{S}^I,\partial\mathcal{S}^J]\left(\hat{T}^J + \text{ intersection terms }\right) \tag{6}$$

where intersection terms stands for additional terms that arise if it happens that the loop γ in the quantum state acted on intersects the boundary $\partial\mathcal{S}^J$ exactly at the surface $\mathcal{S}^I$ or if the boundary itself self-intersects at that surface. We will not need the detailed form of these terms for the considerations of this paper.

2.2. Construction of the quantum reference system

With these results in hand, we may now construct a quantum reference system. The problem that we must face to construct a measurement theory for quantum gravity is

[7] In the case that there is an intersection at the surface, the eigenbra's and eigenvalues can still be found, following the method described in [LS91].

how to give a diffeomorphism invariant description of the reference frame and measuring instruments because, as the geometry of spacetime is the dynamical variable we wish to measure, there is no background metric available to use in their description. The key idea is then that a reference frame must be specified by a particular topological arrangement of the matter fields that go into its construction. In the simple model we are considering here it is very easy to do this. As our reference frame is to consist of surfaces, what we need to give to specify the reference frame is a particular topological arrangement of these surfaces.

One way to do this is the following[LS93]. Choose a simplicial decomposition of the three manifold Σ which has N faces, which we may label $\mathcal{F}_I$. For reasons that will be clear in a moment, it is simplest to restrict this choice to simplicial decompositions in which the number of edges is also equal to N. Let me call such a choice $\mathcal{T}$.

Now, for each $\mathcal{T}$ with N faces there is a subspace of the state space $\mathcal{H}_{diffeo}$ which is spanned by basis elements $< \{\gamma, \mathcal{S}\}|$ in which the surfaces $\mathcal{S}_I$ can be put into a one to one correspondence with the faces $\mathcal{F}_I$ of $\mathcal{T}$ so that they have the same topology of the faces of the simplex. We may call this subspace $\mathcal{H}_{\mathcal{T},diffeo}$.

As I will describe in section 4, the preparation of the system is described by putting the system into such a subspace of the Hilbert space associated with an arrangement of the surfaces. Once we know the state is in the subspace $\mathcal{H}_{\mathcal{T},diffeo}$ any measurements of the quantities $\hat{\mathcal{A}}^I$ or $\hat{T}^I$ can be interpreted in terms of areas of the faces $\mathcal{F}^I$ or parallel transports around their edges.

2.3. How do we describe the results of the measurements?

Given a choice of the simplicial manifold $\mathcal{T}$ and the corresponding subspace $\mathcal{H}_{\mathcal{T},diffeo}$ we may now make measurements of the gravitational field. As I will establish in section 4, there will be circumstances in which it is meaningful to say that we have, at some particular time, made a measurement of some commuting subset of the operators $\hat{\mathcal{A}}^I$ and $\hat{T}^I$ which we described above. We may first note that these operators are block diagonal in $\mathcal{H}_{diffeo}$ in that their action preserves the subspaces $\mathcal{H}_{\mathcal{T},diffeo}$. From the commutation relations (6) we may deduce that if we restrict attention to one of these subspaces and these observables there are two maximal sets of commuting operators; we may measure either the N $\hat{\mathcal{A}}^I$ or the N $\hat{T}^I$. This corresponds directly to the fact that the canonical pair of fields in the Ashtekar formalism are the spatial frame field and the self-dual connection.

How are we to describe the results of these measurements? Each gives us N numbers, which comprise partial information that we can obtain about the geometry of spacetime as a result of a measurement based on the quantum reference frame built on $\mathcal{T}$. Now, as in ordinary quantum mechanics, we would like to construct a classical description of the result of such a partial measurement of the system. I will now show that in each case it is possible to do this. What we must do, in each case, is associate to the results of the measurements, a set of classical gravitational fields that are described in an appropriate way by N parameters.

It is simplest to start with the measurements of the areas, which give us a partial measurement of the spatial geometry.

Classical description of the output of the measurements of the areas

The output of a measurement of the N areas will be N rational numbers, a_I, (times the Planck area), each from the discrete series of possible eigenvalues in the spectra of the $\hat{A}^I$. Let us associate to each such set of areas a piecewise flat three geometry $\mathcal{Q}(a)$ that can be constructed as follows. $\mathcal{Q}(a)$ is the Regge manifold constructed by putting together flat tetrahedra according to the topology given by $\mathcal{T}$ such that the areas of the N faces $\mathcal{F}_I$ are given by $a_I l^2_{Planck}$. Since such a Regge manifold is defined by its edge lengths and since we have fixed $\mathcal{T}$ so that the number of its edges is equal to the number of its faces, the N areas $a_I l^2_{Planck}$ will generically determine the N edge lengths.

Note that we are beginning with the assumption that all of the areas are positive real numbers, so the triangle inequalities must always be satisfied by the edge lengths. At the same time, the tetrahedral identities may not be satisfied, in that there is no inequality which restricts the areas of the faces of an individual tetrahedron in $\mathcal{T}$. For example, there exist configurations $\{\gamma, \mathcal{S}\}$ in which the loop only intersects one of the faces, giving that one face a finite area while the remaining faces have vanishing area. Thus, we must include the possibility that $\mathcal{Q}(a)$ contains tetrahedra with flat metrics with indefinite signatures. The emergence of a geometry that, at least when measured on large scales, may be approximated by a positive definite metric must be a property of the classical limit of the theory.

There may also be special cases in which more than one set of edge lengths are consistent with the areas. In which case we may say that the measurement of the quantum geometry leaves us with a finite set of possible classical geometries. There is nothing particularly troubling about this, especially as this will not be the generic case.

Thus, in general, the outcome of each measurement of the N area operators may be describe by a particular piecewise flat Regge manifold, which represents the partial measurement that has been made of the spatial geometry.

Classical description of the output of the measurements of the self-dual parallel transports

What if we measure instead the N Wilson loops, $\hat{T}^I$? The output of such a measurement will be N complex numbers, t^I. Can we associate these with a classical construction? I want to show here that the answer is yes.

Any such classical construction must not involve a spatial metric, as we have made the spatial geometry uncertain by measuring the quantities conjugate to it. So it must be a construction which is determined by N pieces of information about the self-dual part of the spacetime curvature.

Such a construction can be given, as follows. We may construct a dual graph to $\mathcal{T}$ in the natural way by associating to each of its tetrahedra a vertex and to each of its faces, $\mathcal{F}_I$ an edge, called α_I, such that the 4 α_I associated with the faces of a given tetrahedra have one of their end points at the vertex that corresponds to it. As each face is part of two tetrahedra, we know where to put the two end points of each edge, so the construction is completely determined. We may call this dual graph $\Gamma_{\mathcal{T}}$.

Now, to each such graph we may associate a distributional self-dual curvature which is written as follows,

$$F^i_{ab}(x) = \sum_I \int d\alpha^c_I(s)\ \epsilon_{abc}\ \delta^3(x, \alpha_I(s))\ b^i_I \tag{7}$$

which is determined by giving N $SL(2,C)$ algebra elements b^i_I. If we use the non-abelian Stokes theorem [YaA80], we may show that the moduli of the N complex b^i_I are determined by the N complex numbers t_I that were the output of the measurement by

$$\frac{1}{2} Tr e^{\imath b^i_I \tau^i} = cos|b_I| = t_I \tag{8}$$

where τ^i are, of course, the three Pauli matrices. The remaining information about the orientation of the b^i_I is gauge dependent and is thus not fixed by the measurement.

The reader may wonder whether a connection field can be associated with a distributional curvature of the form of (7). The answer is yes, what is required is a Chern-Simon connection with source given by (7). For any such source there are solutions to the Chern-Simon equations, which, however, require additional structure to be fully specified. One particularly simple way to do it, which does not depend on the imposition of a background metric, is the following[LS91, MS93]. Let us give an arbitrary specification of the faces of the dual graph $\mathcal{G}_{\mathcal{T}}$, which I will call $\mathcal{K}_I$. We may note that there is one face of the dual graph for each edge of $\mathcal{T}$, so that their number is also equal to N. Then we may specify a distributional connection of the form,

$$A^i_a(x) = \sum_I \int d^2\mathcal{K}^{bc}_I(\sigma)\epsilon_{abc}\delta^3(x, \mathcal{K}_I(\sigma)) a^i_I \tag{9}$$

where the a^i_I are, again, N Lie algebra elements. It is then not difficult to show that the usual relationship between the connection and the curvature holds, in spite of their distributional form and that the b^i_I's may be expressed in terms of the a^i_I's. For details of this the reader is referred to [MS93].

Thus, we have shown that to each measurement of the N Wilson loops of the self-dual connection, we can also associate a classical geometry, whose construction is determined by N pieces of information (in this case complex numbers.) The result may be thought of as a partial determination of the geometry of a spacetime Regge manifold. If we add a dimension, and allow time to be discrete than the construction we have just given can be thought of as a spatial slice through a four dimensional Regge manifold. In that case, each edge in our construction becomes a face in the four dimensional construction and, as in the Regge case, the curvature is seen to be distributional with support on the faces. However, in this case it is only the self-dual part of the curvature that is given, because its measurement makes impossible the measurement of any conjugate information. In fact, a complete construction of a Regge-like four geometry can be given along these lines, for details, see [MS93].

2.4. The spatially diffeomorphism invariant inner product

In all of the constructions so far given, the inner product has played no role because we have been expressing everything in terms of the eigenbras of the operators in a particular basis, which are the $< \{\gamma, \mathcal{S}\}|$. However, as in ordinary quantum mechanics, a complete description of the measurement theory will require an inner product. The complete specification of the inner product must be done at the level of the physical states, which requires that we take into account that the states are solutions to the Hamiltonian constraint. This problem, which I would like to claim is essentially equivalent to the problem of time, is the subject of the next section. But it is interesting and, as we shall see, useful to see how much can be determined about the inner product at the level of spatially diffeomorphism invariant states.

Now, in order to determine the inner product at the diffeomorphism invariant level we should be able to write the reality conditions that our classical diffeomorphism invariant observables satisfy. For the $\hat{T}^I$ this is an unsolved problem, these operators are complex, but must satisfy reality conditions which are determined by the reality conditions on the Ashtekar connections. To solve this problem it will be necessary to adjoin additional diffeomorphism invariant operators to the $\hat{T}^I$ and $\hat{\mathcal{A}}^I$ in order to enable us to write down a complete star algebra of diffeomorphism invariant observables.

However, the reality conditions the area operators satisfy are very simple: they must be real. As a result we may ask what restrictions we may put on the inner product such that

$$\hat{\mathcal{A}}^{I\dagger} = \hat{\mathcal{A}}^I? \tag{10}$$

To express this we must introduce characteristic kets, which will be denoted $|\{\gamma, \mathcal{S}\}>$. They are defined so that

$$\Psi_{\{\gamma,\mathcal{S}\}}[\{\gamma,\mathcal{S}\}'] \equiv < \{\gamma,\mathcal{S}\}'|\{\gamma,\mathcal{S}\}> = \delta_{\{\gamma,\mathcal{S}\}\{\gamma,\mathcal{S}\}'} \tag{11}$$

Here the meaning of the delta is follows. Fix a particular, but arbitrary set of surfaces and loops $(\gamma, \mathcal{S})$ within the diffeomorphism equivalence class $\{\gamma, \mathcal{S}\}$. Then the $\delta_{\{\gamma,\mathcal{S}\}\{\gamma,\mathcal{S}\}'}$ is equal to one if and only if there is an element $\mathcal{S}', \gamma'$ of the equivalence class $\{\gamma, \mathcal{S}\}'$ such that a) for every two form F_{ab} on Σ, $\int_{\mathcal{S}'} F = \int_{\mathcal{S}} F$ and b) for every connection A_a^i on Σ and every component $\gamma_{\mathcal{I}}$ of γ (and similarly for γ'), $T[\gamma'_{\mathcal{I}}] = T[\gamma_{\mathcal{I}}]$. If the condition is not satisfied then $\delta_{\{\gamma,\mathcal{S}\}\{\gamma,\mathcal{S}\}'}$ is equal to zero.

Let me denote the diffeomorphism invariant inner product by specifying the adjoint map from kets to bras,

$$< \{\gamma, \mathcal{S}\}^\dagger| \equiv |\{\gamma, \mathcal{S}\}>^\dagger . \tag{12}$$

It is then straightforward to show that the condition (10) that the N operators must be hermitian restricts the inner product so that, in the case that the loops γ have no intersections with each other,

$$< \{\gamma, \mathcal{S}\}^{\dagger}| =< \{\gamma, \mathcal{S}\}|. \tag{13}$$

3. Physical observables and the problem of time

In this section I would like to describe an operational approach to the problem of time in quantum cosmology, which is based on the point of view that time is no more and no less than that which is measured by physical clocks. The general idea we will pursue is to couple general relativity to a matter field whose behavior makes it suitable for use as a clock. One then turns the Hamiltonian constraint equations into evolution equations that proscribe how spatially diffeomorphism invariant quantities evolve according to the time measured by this physical clock.

We will then try to build up the physical theory with the clock from the spatially diffeomorphism invariant theory for the gravitational and matter degrees of freedom, in which the clock has been left out. To do this we will need to assume several things about the diffeomorphism invariant theory, which are motivated by the results of the last section.

a) In the loop representation we have the complete set of solutions to the spatial diffeomorphism constraints coupled to more or less arbitrary matter fields. Given some choice of matter fields, which I will denote generically by ϕ, I will write the general spatially diffeomorphism invariant state by $\Psi[\{\gamma, \phi\}]_{diffeo}$, where the brackets $\{...\}$ mean spatial diffeomorphism equivalence class. The reference frame fields discussed in the previous section are, for the purpose of simplifying the formulas of this section, included in the ϕ. However, the fields that represent the clock *are not* to be included in these diffeomophism invariant states. As in the previous section, the Hilbert space of diffeomorphism invariant states will be denoted $\mathcal{H}_{diffeo}$.

b) I will assume that an algebra of finite and diffeomorphism invariant operators, called $\mathcal{A}_{diffeo}$, is known on $\mathcal{H}_{diffeo}$. The idea that spatially diffeomorphism invariant operators are finite has become common in the loop representation, where there is some evidence for the conjecture that diffeomorphism invariance requires finiteness [ARS92, CR93, LS93, LS91]. The area and parallel transport operators discussed in the previous section are examples of such operators. I will use the notation $\hat{\mathcal{O}}^I_{diffeo}$ to refer to elements of $\mathcal{A}_{diffeo}$, where I is an arbitrary index that labels the operators.

c) I will assume also that the classical diffeomorphism invariant observables $\mathcal{O}^I_{diffeo}$ that correspond to the quantum $\hat{\mathcal{O}}^I_{diffeo}$ are known. This lets us impose the reality conditions on the algebra, as described in [AA91, AI92, CR91].

d) Finally, I assume that the inner product $< | >_{diffeo}$ on $\mathcal{H}_{diffeo}$ has been determined from the reality conditions satisfied by an appropriate subset of the $\hat{O}^I_{diffeo}$.

3.1. A classical model of a clock

I will now introduce a new scalar field, whose value will be defined to *be* time. It will be called the clock field and written $T(x)$, and it will be assumed to have the unconventional dimensions of time. Conjugate to it we must have a density, which has dimensions of energy density,which will be denoted $\tilde{\mathcal{E}}(x)$ such that[8],

$$\{\tilde{\mathcal{E}}(x), T(y)\} = -\delta^3(x,y). \tag{14}$$

To couple these fields to gravity we must add appropriate terms to the diffeomorphism and Hamiltonian constraints. I will assume that $T(x)$ is a free massless scalar field, so that

$$\mathcal{C}(x) = \frac{1}{2\mu}\tilde{\mathcal{E}}^2 + \frac{\mu}{2}\tilde{\tilde{q}}^{ab}\partial_a T \partial_b T + \mathcal{C}_{grav}(x) + \mathcal{C}_{matter}(x), \tag{15}$$

where $\mathcal{C}_{grav}(x)$ and $\mathcal{C}_{matter}(x)$ are, respectively, the contributions to the Hamiltonian constraint for the gravitational field and the other matter fields and μ is a constant with dimensions of *energy density.* Note that the form of (16) is dictated by the fact that in the Ashtekar formalism the Hamiltonian constraint is a density of weight two.

Similarly, the diffeomorphism constraint becomes

$$\mathcal{D}(v) = \int_\Sigma v^a(\partial_a T)\tilde{\mathcal{E}} + \mathcal{D}_{grav}(v) + \mathcal{D}_{matter}(v) \tag{16}$$

The reader may check that these constraints close in the proper way.

We may note that because there is no potential term for the clock field we have a constants of motion,

$$\mathcal{E} \equiv \int_\Sigma d^3x \tilde{\mathcal{E}}(x). \tag{17}$$

This generates the symmetry $T(x) \rightarrow T(x) + \text{constant}$. It is easy to verify explicitly that

$$\{\mathcal{C}(N), \mathcal{E}\} = \{\mathcal{D}(v), \mathcal{E}\} = 0, \tag{18}$$

(where $\mathcal{C}(N) \equiv \int N\mathcal{C}$) .

We would now like to chose a gauge in which the time slicing of the spacetime is made according to surfaces of constant T. We thus choose as a gauge condition.

$$\partial_a T(x) = 0. \tag{19}$$

This may be solved by setting $T(x) = \tau$, where τ will be taken to be the time parameter. We may note that the condition that the evolution follows surfaces of constant $T(x)$ fixes the lapse because,

$$(\dot{\partial_a T})(x) = \{\mathcal{C}(N), \partial_a T(x)\} = \partial_a(N(x)\tilde{\mathcal{E}}(x)) = 0 \tag{20}$$

Thus, our gauge condition can only be maintained if

$$N(x) = \frac{c}{\tilde{\mathcal{E}}(x)}. \tag{21}$$

[8]We adopt the convention that the delta function is a density of weight one on its first entry. Densities will usually, but not always, be denoted by tildes.

where c is an arbitrary constant.

One way to say what this means is that all but one of the infinite number of Hamiltonian constraints have been broken by imposing the gauge condition (19). The one remaining component of the Hamiltonian constraint is the one that satisfies (21). However, since we must have eliminated the nonconstant piece of $T(x)$ by the gauge fixing (19) we must solve the constraints which have been so broken to eliminate the fields which are conjugate to them. Thus, all but one of the degrees of freedom in $\tilde{\mathcal{E}}(x)$ must be eliminated by solving the Hamiltonian constraint. The one which is kept independent can be taken to be the global constant of motion $\mathcal{E}$ defined by (17). Up to this one overall degree of freedom, the local variations in $\mathcal{E}(x)$ must be fixed by solving the Hamiltonian constraint locally, which gives us[9],

$$\tilde{\mathcal{E}}(x) = \sqrt{-2\mu[\mathcal{C}_{grav}(x) + \mathcal{C}_{matter}(x)]} \tag{22}$$

Note that, to keep the global quantity $\mathcal{E}$ independent, we should solve this equation at all but one, arbitrary, point of Σ.

The idea is then to reduce the phase space and constraints so that the local variations in $T(x)$ and $\tilde{\mathcal{E}}(x)$ are eliminated, leaving only the global variables τ and $\mathcal{E}$. We may note that

$$\{T(x), \mathcal{E}\} = 1. \tag{23}$$

so that the reduced Poisson bracket structure must give

$$\{\tau, \mathcal{E}\} = 1 \tag{24}$$

while the Poisson brackets of the gravitational and matter fields remain as before. After the reduction, we then have one remaining Hamiltonian constraint, which is

$$\begin{aligned} \mathcal{C}_{g.f.} = \mathcal{C}(c/\tilde{E}) &= \frac{c}{2\mu}\mathcal{E} + c\int \frac{d^3x}{\mathcal{E}(x)}\left(\mathcal{C}_{grav}(x) + \mathcal{C}_{matter}(x)\right) \\ &= \frac{c}{2\mu}\mathcal{E} + \frac{c}{\sqrt{2\mu}}\int d^3x\sqrt{-[\mathcal{C}_{grav}(x) + \mathcal{C}_{matter}(x)]} \end{aligned} \tag{25}$$

By (24) $\mathcal{C}_{g.f.}$ generates reparametrization of the global time variable τ. The effect of the reduction on the diffeomorphism constraint is simply to eliminate the time field so that

$$\mathcal{D}(v)_{g.f.} = \mathcal{D}_{grav}(v) + \mathcal{D}_{matter}(v) \tag{26}$$

We may note that the reduced Hamiltonian constraint is invariant under diffeomorphisms and hence commutes with the reduced diffeomorphism constraint.

To summarize, the gauge fixed theory is based on a phase space which consists of the original gravitational and matter phase space, to which the two conjugate degrees of freedom τ and

[9] We make here a choice in taking the positive square root, which is to restrict attention to a subspace of the original phase space. This choice, which we carry through as well in the quantum theory, is the equivalent of a positive frequency condition.

$\mathcal{E}$ have been added. The diffeomorphism constraint remains the original one while there remains one Hamiltonian constraint given by (25).

It is now straightforward to construct physical operators. They must be of the form

$$\mathcal{O} = \mathcal{O}[A, E, \phi, \tau, \mathcal{E}] \tag{27}$$

where A, E are the canonical variables that describe the gravitational field and ϕ stands for any other matter fields. The diffeomorphism constraints imply that

$$\mathcal{O}[A, E, \phi, \tau, \mathcal{E}] = \mathcal{O}[\{A, E, \phi\}, \mathcal{E}, \tau] \tag{28}$$

where the brackets $\{...\}$ indicate that the observable can depend only on combinations of the gravitational and matter fields that are spatially diffeomorphism invariant. The requirement that the observable commute with the reduced Hamiltonian constraint gives us

$$\frac{d\mathcal{O}[A, E, \phi, \tau, \mathcal{E}]}{d\tau} = \sqrt{2\mu}\left\{\int d^3x \sqrt{-[\mathcal{C}_{grav}(x) + \mathcal{C}_{matter}(x)]}, \mathcal{O}[A, E, \phi, \tau, \mathcal{E}]\right\}. \tag{29}$$

Thus, we have achieved the following result concerning physical observables:

For every spatially diffeomorphism invariant observable $\mathcal{O}[\{A, E, \phi\}]_{diffeo}$ which is a function of the gravitational and matter fields (but not the clock degrees of freedom) there is a physical observable whose expression in the gauge given by (19) and (21) is the two parameter family of diffeomorphism invariant observables[10], of the form $\mathcal{O}[\{A, E, \phi\}, \tau, \mathcal{E}]_{g.f.}$ which solves (29) subject to the initial condition that

$$\mathcal{O}[\{A, E, \phi\}, \tau = 0, \mathcal{E}]_{g.f.} = \mathcal{O}[\{A, E, \phi\}]_{diffeo} \tag{30}$$

By construction, we may conclude that the observable $\mathcal{O}[\{A, E, \phi\}, \tau]_{g.f.}$ is the value of the diffeomorphism invariant function $\mathcal{O}[\{A, E, \phi\}]_{diffeo}$ evaluated on the surface $T(x) = \tau$ of the spacetime gotten by evolving the constrained initial data $\{A, E, \phi, T = 0\}$.

Now, $\mathcal{O}[\{A, E, \phi\}, \tau, \mathcal{E}]_{g.f.}$ is the value of a physical observable only in the gauge picked by (19) and (21)). However, once we know the value of any observable in a fixed gauge we may extend it to a fully gauge invariant observable. To do so we look for a gauge invariant function[11] $\mathcal{O}[A, E, \phi, T(x), \mathcal{E}(x)]_{Dirac}$ that commutes with the full diffeomorphism and Hamiltonian constraints, with arbitrary lapses N, that has the same physical interpretation as our gauge fixed observable $\mathcal{O}[\{A, E, \phi\}, \tau, \mathcal{E}]_{g.f.}$. This means that

$$\mathcal{O}[A, E, \phi, T(x), \tilde{\mathcal{E}}(x)]_{Dirac}\big|_{T(x)=\tau, \tilde{\mathcal{E}}(x)=\sqrt{-2\mu[\mathcal{C}_{grav}(x)+\mathcal{C}_{matter}(x)]}} = \mathcal{O}[A, E, \phi, \tau, \mathcal{E}]_{g.f.}. \tag{31}$$

Once we have one such physical observable, we may follow Rovelli [CRT91, CRM91] and construct a one parameter family of physical observables called "evolving constants of motion". These are fully gauge invariant functions on the phase space that, for each τ tell

[10]The subscript $g.f.$ will be used throughout this paper to refer to observables that commute with the full spatial diffeomorphism constraints but only the gauge fixed Hamiltonian constraint.

[11]The subscript *Dirac* will always refer to an observable that commutes with the full set of constraints without gauge fixing.

us the value of the spatially diffeomorphism invariant observable $\mathcal{O}[\{A,E,\phi\}]_{diffeo}$ on the surface $T(x)=\tau$ as a function of the data of the initial surface. That, is for each τ we seek a function $\mathcal{O}'[A,E,\phi,T(x),\mathcal{E}(x)](\tau)$ that commutes with the Hamiltonian constraint for all N (with τ taken as a parameter and not a function on the phase space) and which satisfies

$$\mathcal{O}'[A,E,\phi,T(x),\tilde{\mathcal{E}}(x)](\tau)|_{T(x)=0,\tilde{\mathcal{E}}(x)=\sqrt{-2\mu[\mathcal{C}_{grav}(x)+\mathcal{C}_{matter}(x)]}} = \mathcal{O}[A,E,\phi,\tau,\mathcal{E}]_{g.f.}. \quad (32)$$

3.2. Quantization of the theory with the time field

We would now like to extend the result of the previous section to the quantum theory. To do this we must introduce an appropriate representation for the clock fields and construct and impose the diffeomorphism and Hamiltonian constraint equations.

We will first construct the quantum theory corresponding to the reduced classical dynamics that follows from the gauge fixing (19) and (21). After this we will discuss the alternative possibility, which is to construct the physical theory through Dirac quantization in which no gauge fixing is done.

In the gauge fixed quantization the states will be taken to be functions $\Psi[\gamma,\phi,\tau]$ so that

$$\hat{\tau}\Psi[\gamma,\phi,\tau] = \tau\Psi[\gamma,\phi,\tau], \quad (33)$$

$$\hat{\mathcal{E}}\Psi[\gamma,\phi,\tau] = -\imath\hbar\frac{\partial}{\partial\tau}\Psi[\gamma,\phi,\tau] \quad (34)$$

and all the other defining relations are kept. The space of these states, prior to the imposition of the remaining constraints, will be called $\mathcal{H}_{reduced}$.

We now apply the reduced diffeomorphism constraints (26). The result is that the states be functions of diffeomorphism equivalence classes of their arguments, so that

$$\mathcal{D}(v)_{g.f.}\Psi[\gamma,\phi,\tau]=0 \Rightarrow \Psi[\gamma,\phi,\tau]=\Psi[\{\gamma,\phi\},\tau] \quad (35)$$

where, again, the brackets indicate diffeomorphism equivalence classes.

We may then apply the reduced Hamiltonian constraint (25). By (34) this implies that, formally

$$\frac{\imath\hbar\partial\Psi[\{\gamma,\phi\},\tau]}{\partial\tau} = \int d^3x\sqrt{-2\mu[\hat{\mathcal{C}}_{grav}(x)+\hat{\mathcal{C}}_{matter}(x)]}\,\Psi[\{\gamma,\phi\},\tau] \quad (36)$$

As this is the fundamental equation of the quantum theory, we must make some comments on its form. First, and most importantly, as the Hamiltonian constraint involves operator products, this equation must be defined through a suitable regularization procedure. Secondly, to make sense of this equation requires that we define an operator square root. Both steps must be done in such a way that the result is a finite and diffeomorphism invariant operator. That is, for this approach to quantization to work, it must be possible to regulate the gravitational and matter parts of the Hamiltonian constraint in such a way that the limit

$$\lim_{\epsilon\rightarrow 0}\int d^3x\sqrt{-2\mu[\hat{\mathcal{C}}^{\epsilon}_{grav}(x)+\hat{\mathcal{C}}^{\epsilon}_{matter}(x)]} = \hat{\mathcal{W}} \quad (37)$$

(where the ϵ's denote the regulated operators) exists and gives a well defined (and hence finite and diffeomorphism invariant) operator $\hat{\mathcal{W}}$ on the space of spatially diffeomorphism invariant states of the gravitational and matter fields.

It may seem that to ask that it be possible to both define a good regularization procedure and define the operator square root is to be in danger of being ruled out by the "two miracle" rule: it is acceptable practice in theoretical physics to look forward to the occurrence of one miracle, but to ask for two is unreasonable. However, recent experience with constructing diffeomorphism invariant operators in the loop representation of quantum gravity suggests the opposite: these two problems may be, in fact, each others solution. Perhaps surprisingly, what has been found is that the only operators which have been so far constructed as finite, diffeomorphism invariant operators involve operator square roots.

The reason for this is straightforward. In quantum field theory operators are distributions. In the context of diffeomorphism invariant theories, distributions are densities. As a result, there is an intrinsic difficulty with defining operator products in a diffeomorphism invariant theory through a renormalization procedure. Any such procedure must give a way to define the product of two distributions, with the result being another distribution. But this means that the procedure must take a geometrical object which is formally a density of weight two, and return a density of weight one. What happens as a result is that there is a grave risk of the regularization procedure breaking the invariance under spatial diffeomorphism invariance, because the missing density weight ends up being represented by functions of the unphysical background used in the definition of the renormalization procedure. Many examples are known in which exactly this happens[LS91].

However, it turns out that in many cases the square root of the product of two distributions can be defined as another distribution without ambiguity due to this problem of matching density weights. It is, indeed, exactly this fact that makes it possible to define the area operators I described in the previous section, as well as other operators associated with volumes of regions and norms of one form fields[ARS92, LS91].

I do not know whether the same procedures that work in the other cases work to make the limit (37) exist. We may note, parenthetically, that if $\hat{\mathcal{W}}$ can be defined as an operator on diffeomorphism invariant states in the context of a separable Hilbert structure, the problem of the finiteness of quantum gravity will have been solved. I will assume here that the problem of constructing a regularization procedure such that this is the case can be solved, and go on.

Assuming then the existence of $\hat{\mathcal{W}}$, we may call the space of solutions to (35) and (36) $\mathcal{H}_{g.f}$, where the subscript denotes again that we are working with the gauge fixed quantization. We will shortly be discussing the inner product on this space. For the present, the reader may note that once $\hat{\mathcal{W}}$ exists, the problem of finding states that solve the reduced Hamiltonian constraint is essentially a problem of ordinary quantum mechanics. For example, there will be solutions of the form

$$\Psi[\{\gamma, \phi\}, \tau] = \Phi[\{\gamma, \phi\}] e^{-\imath\omega\tau} \tag{38}$$

This will solve (36) if $\Phi[\{\gamma,\phi\}]$ is an eigenstate of $\hat{\mathcal{W}}$, so that

$$\hat{\mathcal{W}}\Phi = \hbar\omega\Phi \tag{39}$$

More generally, given any diffeomorphism invariant state $\Psi[\{\gamma,\phi\}] \in \mathcal{H}_{diffeo}$, which is a function of the gravitational and matter fields alone, there is a physical state, $\Psi[\{\gamma,\phi\},\tau] \in \mathcal{H}_{g.f}$ in our gauge fixed quantization which is the solution to (36) with the initial conditions

$$\Psi[\{\gamma,\phi\},\tau]_{\tau=0} = \Psi[\{\gamma,\phi\}] \tag{40}$$

Thus what we have established is that there is a map

$$\Lambda : \mathcal{H}_{diffeo} \rightarrow \mathcal{H}_{g.f.} \tag{41}$$

in which every diffeomorphism invariant state of the gravitational and matter fields is taken into its evolution in terms of the clock fields. Furthermore, there is an inverse map,

$$\Theta : \mathcal{H}_{g.f.} \rightarrow \mathcal{H}_{diffeo} \tag{42}$$

which is defined by evaluating the physical state at $\tau = 0$.

3.3. The operators of the gauge fixed theory

The physical operators in the gauge fixed quantum theory may be found analogously to the classical observables of the gauge fixed theory. A physical operator is an operator on $\mathcal{H}_{g.f.}$, which may be written $\hat{\mathcal{O}}[\hat{A},\hat{E},\hat{\phi},\hat{\tau},\hat{\mathcal{E}}]$. The requirement that it commute with the reduced diffeomorphism constraints restricts it to be of the form $\hat{\mathcal{O}}[\{\hat{A},\hat{E},\hat{\phi}\},\hat{\tau},\hat{\mathcal{E}}]$. The requirement that it commute with the reduced Hamiltonian constraint becomes the evolution equation,

$$\imath\hbar\frac{d\hat{\mathcal{O}}[\hat{A},\hat{E},\hat{\phi},\tau,\hat{\mathcal{E}}]}{d\tau} = \left[\hat{\mathcal{W}},\hat{\mathcal{O}}[\hat{A},\hat{E},\hat{\phi},\tau,\hat{\mathcal{E}}]\right]. \tag{43}$$

Using this equation we may find a physical operator that corresponds to every spatially diffeomorphism invariant operator on $\mathcal{H}_{diffeo}$, which depends only on the non-clock fields. Given any such operator, $\hat{\mathcal{O}}[\{\hat{A},\hat{E},\hat{\phi}\}]_{diffeo}$ we may construct an operator on $\mathcal{H}_{g.f.}$ which we denote $\hat{\mathcal{O}}[\{\hat{A},\hat{E},\hat{\phi}\},\tau]_{g.f.}$ which solves (43) subject to the initial condition

$$\hat{\mathcal{O}}[\{\hat{A},\hat{E},\hat{\phi}\},\tau=0]_{g.f.} = \hat{\mathcal{O}}[\{\hat{A},\hat{E},\hat{\phi}\}]_{diffeo} \tag{44}$$

We may, indeed, solve (43) to find that

$$\hat{\mathcal{O}}[\{\hat{A},\hat{E},\hat{\phi}\},\tau]_{g.f.} = e^{-\imath\hat{\mathcal{W}}\tau/\hbar}\hat{\mathcal{O}}[\{\hat{A},\hat{E},\hat{\phi}\}]_{diffeo}e^{+\imath\hat{\mathcal{W}}\tau/\hbar}. \tag{45}$$

3.4. The physical interpretation and inner product of the gauge fixed theory

In the classical theory, the physical observables $\mathcal{O}[\{A, E, \phi\}, \tau]_{g.f.}$ were found to correspond to the values of diffeomorphism invariant functions of the non-clock fields on the surface defined by the gauge condition (19) and the value of the time parameter τ. As they satisfy the analogous quantum equations we would like to interpret the corresponding quantum operators that solve (43) and (44) in the same way. That is, we will take $\hat{\mathcal{O}}[\{\hat{A}, \hat{E}, \hat{\phi}\}, \tau]_{g.f.}$ to be the operator that measures the diffeomorphism invariant quantity $\hat{\mathcal{O}}[\hat{E}, \hat{A}, \hat{\phi}]_{diff}$ after a physical time τ.

Once this interpretation is fixed, there is a natural choice for the physical inner product. The idea, advocated by Ashtekar [AA91], is that the physical inner product is to be picked to satisfy the reality conditions for a large enough set of physical observables. The difficult part of the definition is the meaning of "large enough", but study of a number of examples shows that large enough means a complete set of commuting operators, and an equal number of operators conjugate to them [AA91, AA92, RT92]. Now, as in ordinary quantum field theory, it is very unlikely that any two operators defined at different physical times commute. Thus, we may postulate that the largest set of operators for which reality conditions can be imposed for the physical theory are two conjugate complete sets defined at a single moment of physical time.

Thus, we could define an inner product by imposing the reality conditions for a complete set of operators $\hat{\mathcal{O}}[\{\hat{A}, \hat{E}, \hat{\phi}\}, \tau]_{g.f.}$ at any physical time τ. Now, the nicest situation would be if the resulting inner products were actually independent of τ. However, there are reasons, some of which are discussed below, to believe that this may not be realized in full quantum gravity. If this is the case then the most natural assumption to make is that the physical inner product must be determined by a complete set of operators at the initial time, $\tau = 0$, as that will correspond to the time of preparation of the physical system.

However, by (44) we see that to impose the reality conditions on the operators $\hat{\mathcal{O}}[\{\hat{A}, \hat{E}, \hat{\phi}\}, \tau]_{g.f.}$ at $\tau = 0$ is to simply impose the diffeomorphism invariant reality conditions for the non-clock fields. Thus, we may propose that for any two physical states Ψ and Ψ' in $\mathcal{H}_{g.f.}$

$$< \Psi|\Psi' >_{g.f.} = < \Theta \circ \Psi|\Theta \circ \Psi' >_{diffeo} \tag{46}$$

As a result, we may conclude that the physical expectation value of the operator that measures the diffeomorphism invariant quantity $\mathcal{O}[\{A, E, \phi\}]_{diffeo}$ at the time τ in the state $\Psi[\{\gamma, \phi\}, \tau]$ in $\mathcal{H}_{g.f.}$ is given by

$$< \Psi|\hat{\mathcal{O}}[\{\hat{A}, \hat{E}, \hat{\phi}\}, \tau]_{g.f.}|\Psi >_{g.f.} = < \Theta \circ \Psi|\Theta \circ \left(\hat{\mathcal{O}}[\{\hat{A}, \hat{E}, \hat{\phi}\}, \tau]_{g.f.}|\Psi >\right)_{diffeo} \tag{47}$$

3.5. A word about unitarily

The reader may notice that in fixing the physical inner product there was one condition I might have imposed, but did not. This was that the operator $\hat{\mathcal{W}}$ that generates evolution for the non-clock fields be hermitian. The reason for this is one aspect of the conflict between the notions of time in quantum theory and general relativity. From the point of view of quantum theory, it is natural to assume that the time evolution operator is unitary. However, this means that all physical states of the form $\Psi[\{\gamma,\phi\},\tau]$ exist for all physical clock times τ. This directly contradicts the situation in classical general relativity, in which for every set of initial data which solves the constraints for compact Σ (and which satisfies the positive energy conditions) there is a time τ after which the spacetime has collapsed to a final singularity so that no physical observable could be well defined.

Furthermore, we may note that we may not be free to choose the physical inner product such that $\hat{\mathcal{W}}$ is hermitian. The physical inner product has already been restricted by the reality conditions applied to a certain set of observables of the diffeomorphism invariant theory. As $\hat{\mathcal{W}}$ is to be defined through a limit of a regularization procedure, it is probably best to fix the inner product first and then define the limit inside this inner product space[12]. However, then it is not obvious that the condition that $\hat{\mathcal{W}}$ be hermitian will be consistent with the conditions that determine the inner product.

How a particular formulation of quantum gravity resolves these conflicts is a dynamical problem. This is, indeed, proper, as the evolution operator for quantum gravity could be unitary only if the quantum dynamics avoided complete gravitational collapse in every circumstance, and whether this is the case or not is a dynamical problem. The implications of this situation will be the subject of section 6.

3.6. The physical quantum theory without gauge fixing

As in the classical theory, once we have the gauge fixed theory it is easier to see how to construct the theory without gauge fixing. Here I will give no technical details, but only sketch the steps of the construction.

In the Dirac approach, we first construct the kinematical state space, which consists of all states with a general dependence on the variables, of the form $\Psi[\gamma,\phi,T(x)]_{Dirac}$. Instead of (33) and (34) we have the defining relations

$$\hat{T}(x)\Psi[\gamma,\phi,T(x)]_{Dirac} = T(x)\Psi[\gamma,\phi,T(x)]_{Dirac} \tag{48}$$

and

$$\hat{\mathcal{E}}(x)\Psi[\gamma,\phi,T(x)]_{Dirac} = -\imath\hbar\frac{\delta}{\delta T(x)}\Psi[\gamma,\phi,T(x)]_{Dirac}. \tag{49}$$

The physical state space, $\mathcal{H}_{Dirac}$ then consists of the subspace of states that satisfy the full set of constraints,

$$\hat{\mathcal{D}}(v)\Psi_{Dirac} = 0 \tag{50}$$

[12]Note that this is different from the problem of finding the kernel of the constraints, for which it is sufficient to define the limit in a pointwise topology, as was done in [JS88, RS88]. There we were content to let the limit be undefined on the part of the state space not in the kernel. As we now want to construct the whole operator we probably need an inner product to control the limit.

and

$$\lim_{\epsilon \to 0} \hat{\mathcal{C}}^{\epsilon}(N)\Psi_{Dirac} = 0 \tag{51}$$

for all v^a and N. As in the gauge fixed formalism, we will be interested only in those solutions that arise from initial data of the non-clock fields, so that they can represent states prepared at an initial clock time. Thus, we will be interested in the subspace of Dirac states such that

$$\Psi[\{\gamma, \phi, T(x) = 0\}]_{Dirac} = \Psi[\{\gamma, \phi\}]_{diffeo}. \tag{52}$$

is normalizable in an appropriate inner product that gives a probability measure to the possible preparations of the system we can make at the initial time.

There is a complication that arises in the case of the full constraints, because (51) is a second order equation in $\delta/\delta T(x)$. This is the familiar problem of the doubling of solutions arising from the Klein-Gordon like form of the full Hamiltonian constraint. In the gauge fixed quantization we studied in section 3.2-3.4 this problem did not arise because the reduced Hamiltonian constraint was first order in the derivatives of the reduced time variable τ.

However, there is a way to deal with this problem in the full, Dirac, quantization, because we have the constant of motion $\mathcal{E}$ defined by (17). We can use this to impose a positive frequency condition on the physical states. Thus, using $\hat{\mathcal{E}}$ we can split the Hilbert space $\mathcal{H}_{Dirac}$, to be the direct product of two subspaces, $\mathcal{H}^{\pm}_{Dirac}$, where $\mathcal{H}^{+}_{Dirac}$ is spanned by the eigenstates of $\hat{\mathcal{E}}$ whose eigenvalues have positive real part, and $\mathcal{H}^{-}_{Dirac}$ is spanned by the eigenstates of $\hat{\mathcal{E}}$ with eigenvalues with negative real part. Associated to this splitting we have projection operators $P^{\pm}$ that project onto each of these subspaces[13]. From now on, we will restrict attention to states and operators in the positive frequency part of the physical Hilbert space.

As in the gauge fixed case we thus define a map
$\Theta : \mathcal{H}^{+}_{Dirac} \rightarrow \mathcal{H}_{diffeo}$ by

$$(\Theta \circ \Psi_{Dirac}) [\{\gamma, \phi\}] = \Psi[\{\gamma, \phi, T(x) = 0\}]_{Dirac} \tag{53}$$

Further, if there is a unique solution to the constraints that satisfies also the positive frequency condition,

$$P^{+}\Psi_{Dirac} = \Psi_{Dirac} \tag{54}$$

there is a corresponding inverse map $\Lambda : \mathcal{H}_{diffeo} \rightarrow \mathcal{H}^{+}_{Dirac}$ which takes each state in $\mathcal{H}_{diffeo}$ into its positive frequency evolution under the full set of constraints.

The operators on this space, which we can call the Dirac operators are as well solutions to the full set of constraints,

$$\left[\hat{\mathcal{D}}(v), \hat{\mathcal{O}}_{Dirac}\right] = 0 \tag{55}$$

$$\lim_{\epsilon \to 0} \left[\hat{\mathcal{C}}^{\epsilon}(N), \hat{\mathcal{O}}_{Dirac}\right] = 0 \tag{56}$$

[13] Note that I have not assumed that $\hat{\mathcal{E}}$ is hermitian, for the reasons discussed in the previous section.

We will impose as well the positive frequency condition,

$$\left[P^{+}, \hat{O}_{Dirac}\right] = 0 \tag{57}$$

which converts (56) from a second order to a first order functional differential equation.

We may then seek to use these equations to extend diffeomorphism invariant operators on $\mathcal{H}_{diffeo}$, which act only on the non-clock degrees of freedom, to positive frequency Dirac operators. That is, given an operator $\hat{\mathcal{O}}[\{\hat{E}, \hat{A}, \hat{\phi}\}]_{diffeo}$ we seek operators of the form $\hat{\mathcal{O}}[\{\hat{E}, \hat{A}, \hat{\phi}, T(x), \hat{\mathcal{E}}\}]_{Dirac}$ which solve (55), (56) and (57) which have the property that

$$\hat{\mathcal{O}}[\{\hat{E}, \hat{A}, \hat{\phi}, T(x) = 0, \hat{\mathcal{E}}\}]_{Dirac} = \hat{\mathcal{O}}[\{\hat{E}, \hat{A}, \hat{\phi}\}]_{diffeo} \tag{58}$$

Furthermore, we can construct quantum analogues of the evolving constants of motion (32). These are corresponding one parameter families of Dirac observables $\hat{\mathcal{O}}'[\{\hat{E}, \hat{A}, \hat{\phi}, T(x) = 0, \hat{\mathcal{E}}\}](\tau)_{Dirac}$ that satisfy (55), (56) and (57) (with τ, again, treated just as a parameter) and the condition

$$\hat{\mathcal{O}}'[\{\hat{E}, \hat{A}, \hat{\phi}, T(x) = 0, \hat{\mathcal{E}}\}](\tau)_{Dirac} = \hat{\mathcal{O}}[\{\hat{E}, \hat{A}, \hat{\phi}, T(x) = \tau, \hat{\mathcal{E}}\}]_{Dirac} \tag{59}$$

As in the classical case, one can relate these operators also to the operators of the gauge fixed theory. However, as there are potential operator ordering problems that come from the operator versions of the substitutions in (32), and as the gauge fixed and Dirac operators act on different state spaces, it is more convenient to make the definition in this way.

We may note that these may not be all of the physical operators of the Dirac theory, as there may be operators that satisfy the constraints and positive frequency condition for which there is no diffeomorphism invariant operator of only the non-clock fields such that (58) holds. But this is a large enough set to give the theory a physical interpretation based on the use of the clock fields.

To finish the construction of the Dirac formulation, we must give the physical inner product. The same argument that we gave in the gauge fixed case leads to the conclusion that we may impose a physical inner product $< | >_{Dirac}$ such that, if Ψ and Φ are two elements of $\mathcal{H}^{+}_{Dirac}$

$$< \Psi | \Phi >_{Dirac} = < \Theta \circ \Psi | \Theta \circ \Phi >_{diffeo} \tag{60}$$

For the reason just stated, this may not determine the whole inner product on $\mathcal{H}^{+}_{Dirac}$, but it is enough to do some physics because we may conclude that in the Dirac formalism the expectation value of the operator that corresponds to measuring the diffeomorphism invariant quantity $\mathcal{O}_{diffeo}$ of the nonclock field a physical time τ after the preparation of the system in the state $|\Psi>$ (which, by definition is in $\mathcal{H}^{+}_{Dirac}$) is

$$< \Psi | \hat{\mathcal{O}}'(\tau)_{Dirac} | \Psi >_{Dirac} = < \Theta \circ \Psi | \Theta \circ \left(\hat{\mathcal{O}}'(\tau)_{Dirac} | \Psi > \right)_{diffeo}. \tag{61}$$

4. Outline of a measurement theory for quantum cosmology

I will now move away from technical problems, and consider the question of how a theory constructed according to the lines of the last two sections could be interpreted physically. In order to give an interpretation of a quantum theory it is necessary to describe what mathematical operations in the theory correspond to preparation of the system and what mathematical operations correspond to measurement. This is the main task that I hope to fulfil here. I should note that I will phrase my discussion entirely in the traditional language introduced by Bohr and Heisenberg concerning the interpretation of quantum mechanics. As we will see, with the appropriate modifications, there is no barrier to using this language in the context of quantum cosmology. However, if the reader prefers a different language to discuss the interpretation of quantum mechanics, whether it be the many worlds interpretation or a statistical interpretation, she will, as in the case of ordinary quantum mechanics, be able to rephrase the language appropriately.

The interpretation of quantum cosmology that I would like to describe is based on the following four principles:

A) The measurement theory must be completely spacetime diffeomorphism invariant. The interpretation must respect the spacetime diffeomorphism invariance of the quantum theory of gravity. Thus, we must build the interpretation entirely on physical states and physical operators.

B) The reference system, by means of which we locate where and when in the universe measurements take place, must be a dynamical component of the quantum matter plus gravity system on which our quantum cosmology is based. This is a consequence of the first principle, because the diffeomorphism invariance precludes the meaningful use of any coordinate system that does not come from the configuration of a dynamical variable.

C) As we are are studying a quantum field theory, any measurement we can make on the system must be a partial measurement. This is an important point whose implications will play a key role in what follows. The argument for it is simple: a quantum field theory has an infinite number of degrees of freedom. Any measurement that we make returns a finite list of numbers. The result is that any measurement made on a quantum field theory can only result in a partial determination of the state of the system.

D) The inner product is to be determined by requiring that a complete set of physical observables for the gravity and matter degrees of freedom satisfy the reality conditions at the initial physical time corresponding to preparation of the state.

For concreteness I will phrase the measurement theory in terms of the particular type of reference frames and clock fields described in the last two sections. However, I will use a language that can refer to either the gauge fixed formalism described in subsections 3.2-3.4 or the Dirac formalism described in subsection 3.6. I will use a general subscript *phys* to refer to the physical states, operators and inner products of either formalism. If one wants to specify the gauge fixed formalism then read *phys* to mean $g.f.$ so that operators

$\hat{\mathcal{O}}(\tau)_{phys.}$ will mean the gauge fixed operators $\hat{\mathcal{O}}[\hat{E}, \hat{A}, \hat{\phi}, \hat{\mathcal{E}}, \tau]_{g.f.}$ defined in subsection 3.3. Alternatively, if one wants to think in terms of the Dirac formalism then read *phys* to mean $Dirac$ everywhere, so that the operators $\hat{\mathcal{O}}(\tau)_{phys.}$ refer to the τ dependent "evolving constants of motion" $\hat{\mathcal{O}}'(\tau)_{Dirac}$. Furthermore, I will always assume that reference is being made to states and operators in the positive frequency subspace of the Dirac subspace. I will use this notation as well in section 6.

Thus, putting together the results of the last two sections, I shall assume, for purposes of illustration, that we have available at least two sets of τ dependent physical observables $\hat{\mathcal{A}}^I(\tau)_{phys}$ and $\hat{T}^I(\tau)_{phys}$ which measure, respectively, the areas of the simplices of the reference frame, and parallel transport around them, at a physical time τ.

However, while I refer to a particular form of clock dynamics and a particular set of observables, I expect that the interpretation given here can be applied to any theory in which the physical states, observables and inner product are related to their diffeomorphism invariant counterparts in the way described in the last section.

Let us now begin with the process of preparing a system for an observation.

4.1. Preparation in quantum cosmology

Let me assume that at time $\tau = 0$ we make a preparation prior to performing some series of measurements on the quantum gravitational field. This means that we put the quantum fields which describe the temporal and spatial reference system into appropriate configurations so that the results of the measurements will be meaningful. There are two parts to the preparation: arrangement of the spatial reference system and synchronization of the physical clocks.

As in ordinary quantum mechanics, we can assume that we, as observers, can move matter around as we choose in order to do this. This certainly does not contradict the assumption that the whole universe including ourselves could be described by the quantum state Ψ for, if it did, we would be simply unable to do quantum cosmology because, *ipso facto* we are in the universe and we do move things such as clocks and measuring instruments around more or less as we please.

In ordinary quantum mechanics the act of preparation may be described by projecting the quantum states of the reference system and measuring instruments into appropriate states, after which the direct product with the system state is taken. In the case of a diffeomorphism invariant theory we cannot do this because there is no basis of the diffeomorphism invariant space $\mathcal{H}_{diffeo}$ whose elements can be written as direct products of matter states and gravity states. Thus, the requirement of diffeomorphism invariance has entangled the various components of the whole system even before any interactions occur.

However, this entanglement does not prevent us from describing in quantum mechanical terms the preparation of the reference system and clock fields. What we must do is describe the preparation by projecting the physical states into appropriate subspaces, every state of which describes a physical situation in which the matter and clock fields have been prepared appropriately.

Let us begin with synchronizing the clocks. We may assume that we are able to synchronize a field of clocks over as large a volume of the universe as we please, or even over the whole universe (if it is compact). The difficulty of doing this is, *a priori* a practical problem, not a problem of principle. So we assume that we may synchronize our clock field so that there is a spacelike surface everywhere on which $T(x) = 0$.

In terms of the formalism, this act of preparing the clocks corresponds to assuming that the state is normalizable in the inner product defined by either (46) or (60). That is, there may be states in either the solution space to the gauge fixed constraints or the Dirac constraints that are not normalizable in the respective inner products defined by the maps to the diffeomorphism invariant states of the non-clock fields. Such states cannot correspond to preparations of the matter and gravitational fields made at some initial time of the clock fields.

Once the clocks are synchronized we can prepare the spatial reference frame. As described in subsections 2.2 and 2.3 we do this by specifying that the N surfaces are arranged as the faces of a simplicial complex $\mathcal{T}$. In the formalism this is described by the statement that the state of the system is to be further restricted to be in a subspace $\mathcal{H}_{phys,\mathcal{T}}$ of $\mathcal{H}_{phys}$.

It is interesting to note that, at least in principle, the preparation of the spatial and temporal reference frames can be described without making any assumption about the quantum state of the gravitational field. Of course, this represents an ideal case, and in practice preparation for a measurement in quantum cosmology will usually involve fixing some degrees of freedom of the gravitational field. But, it is important to note that this is not required in principle. In particular, no assumption need be made restricting the gravitational field to be initially in anything like a classical or semiclassical configuration to make the measurement process meaningful.

This completes the preparation of the spatial and temporal reference frames[14].

4.2. Measurement in quantum cosmology

After preparing the system we may want to wait a certain physical time τ before making a measurement. Let us suppose, for example, that we want to measure the area of one of the

[14]Note that, in a von Neumann type description of the measuring process, we must also include the measuring instruments in the description of the system. These are contained in the dependence of the physical states on additional matter fields in the set ϕ that represent the actual measuring instruments. I will not here go through the details of adopting the von Neumann description of measurement to the present case, but there is certainly no obstacle to doing so. However, as pointed out by Anderson [AAn93] it is necessary to take into account the fact that a real measuring interaction takes a finite amount of physical time. There is also no obstacle to including in the preparation as much information about our own existence as may be desired, for example, there is a subspace of $\mathcal{H}_{phys,\mathcal{T}}$ in which we are alive, awake, all our measurement instruments are prepared and we are in a mood to do an experiment. There is no problem with assuming this and requiring that the system be initially in this subspace. However, as in ordinary quantum mechanics, as long as we do not explicitly make any measurements on ourselves, there is no reason to do this.

surfaces picked out by the spatial reference system at the time τ after the preparation. How are we to describe this? To answer this question we need to make a postulate, analogous to the usual postulates that connect measurements to the actions of operators in quantum mechanics. It seems most natural to postulate the following:

Measurement postulate of quantum cosmology: *The operator that corresponds to the making of a measurement of the spatially diffeomorphism invariant quantity represented by* $\hat{O}_{diffeo} \in \mathcal{H}_{diffeo}$ *at the timhat the clock field reads* τ*, is the physical (meaning either gauge fixed or Dirac) operator* $\hat{O}_{phys}(\tau)$*. Thus, we postulate that the expected value of making a measurement of the quantity* $\mathcal{O}$ *at a time* τ *after the preparation in the physical state* $|\Psi>$ *is given by* $<\Psi|\hat{O}_{phys}(\tau)|\Psi>_{phys}$*.*

It is consistent with this to postulate also that: *the only possible values which may result from a measurement of the physical quantity* $\hat{O}_{phys}(t)$ *are its eigenvalues, which may be found by solving the physical states equation*

$$\hat{O}_{phys}(\tau)|\lambda(\tau)>_{phys}=\lambda(\tau)|\lambda(\tau)>_{phys} \tag{62}$$

inside the physical state space $\mathcal{H}_{phys}$.

Having described how observed quantities correspond to the mathematical expressions of quantum cosmology, we have one more task to fulfil to complete the description of the measurement theory. This is to confront the most controversial part of measurement theory, which is the question of what happens to the quantum state after we make the measurement. In ordinary quantum mechanics there are two points of view about this, depending on whether one wants to employ the projection postulate or some version of the relative state idea of Everett. This choice is usually, but perhaps not necessarily, tied to the philosophical point of view that one holds about the quantum state. If one believes, with Bohr and von Neumann, that the quantum state is nothing physically real, but only represents our information about the system, then there is no problem with speaking in terms of the projection postulate. There is, in this way of speaking, only an abrupt change in the information that we have about the system. Nothing physical changes, i.e. collapse of the wave function is not a physical event or process.

On the other hand, if one wants to take a different point of view and postulate that the quantum state is directly associated with something real in nature, the projection postulate brings with it the well known difficulties such as the question of in whose reference frame the collapse takes place. There are then two possible points of view that may be taken. Some authors, such as Penrose [RP], take this as a physical problem, to be solved by a theory that is to replace, and explain, quantum mechanics. Therefore, these authors want to accept the collapse as being something that physically happens. The other point of view is to keep the postulate that the quantum state is physically real but to give an interpretation of the theory that does not involve the projection postulate. In this case one has to describe measurement in terms of the correlations that are set up during the measurement process between the quantum state of the measuring instrument and the quantum state of the system as a result of their interaction during the measurement.

Of course, these two hypotheses lead, in principle, to different theories as in the second it is possible to imagine doing experiments that involve superpositions of states of the observer, while in the first case this is not possible. Nevertheless, there is a large set of cases in which the predictions of the two coincide. Roughly, these are the cases in which the quantum state, treated from the second point of view, would decohere. Indeed, if one takes the second point of view then some version of decoherence is necessary to recover what is postulated from the first point of view, which is that the observer sees a definite outcome to each experiment.

I do not intend to settle here the problem of which of these points of view corresponds most closely to nature. However, I would like to make two claims which I believe, to some extent, diffuse the conflict. First, I would like to claim that whichever point of view one takes, something like the statement of the projection postulate plays a role. Whether it appears as a fundamental statement of the interpretation, or as an approximate and contingent statement which emerges only in the case of decoherent states or histories, the connection to what real observers see can be described in terms of the projection postulate, or something very much like it. Second, I would like to claim that the situation is not different in quantum cosmology then it is in ordinary quantum mechanics. One can make either choice and, in each case, something like the projection postulate must enter when you discuss the results of real observations (at least as long as one is not making quantum observations on the brain of the observer.)

With these preliminaries aside, I will now state how the projection postulate can be phrased so that it applies in the quantum cosmological case:

Cosmological projection postulate: *Let $\hat{\mathcal{O}}_I(\tau_0)_{phys}$ be a finite set of physical operators that mutually commute and hence correspond to a set of measurements that can be made simultaneously at the time τ_0. Let us assume that the reference system has been prepared so that the system before the measurements is in the subspace $\mathcal{H}_{phys,\mathcal{T}}$. The results of the observations will be a set of eigenvalues $\lambda_I(\tau_0)$. For the purposes of making any further measurements, which would correspond to values of the physical clock field τ for which $\tau > \tau_0$, the quantum state can be assumed to be projected at the physical clock time τ_0 into the subspace $\mathcal{H}_{phys,\mathcal{T},\lambda(\tau_0)} \subset \mathcal{H}_{phys,\mathcal{T}}$ which is spanned by all the eigenvectors of the operators $\hat{\mathcal{O}}_I(\tau_0)_{phys}$ which correspond to the eigenvectors $\lambda_I(t_0)$.*

That is, one is to project the state into the subspace at the time of the measurement, and then continue with the evolution defined by the Hamiltonian constraint.

4.3. Discussion

I would now like to discuss three objections that might be raised concerning the application of the projection postulate to quantum cosmology. Again, let me stress that my goal here is not to argue that one must take Bohr's point of view over that of the other interpretations. I only want to establish that if one is happy with Bohr in ordinary quantum mechanics one can continue to use his point of view in quantum cosmology. The only strong claim I want to make is that the statement sometimes made, that one is required to give up Bohr's point of view when one comes to quantum cosmology, is false.

First objection: *Bohr explicitly states that the measurement apparatus must be described classically, which requires that it be outside of the quantum system being studied.* I believe that this represents a misunderstanding of Bohr which, possibly, comes from combining what Bohr did write with an assumption that he did not make, which is that the quantum state is in one to one correspondence with something physically real. For Bohr to have taken such a realistic point of view about the quantum state would have been to directly contradict his fundamental point of view about physics, which is that it does not involve any claim to a realistic correspondence between nature and either the mathematics or the words we use to represent the results of observations we make. Instead, for Bohr, physics is an extension of ordinary language by means of which we describe to each other the results of certain activities we do. Bohr takes it as given that we must use classical language to describe the results of our observations because that is what real experimentalists do. Perhaps the weakest point of Bohr is his claim that it is necessary that we do this, but even if we leave aside his attempts to establish that, we are still left with the fact that up to do this day the only language that we actually do use to communicate with each other what happens when we do experimental physics uses certain classical terms.

Furthermore, rather than insisting that the measuring instrument is outside of the quantum system, Bohr insists repeatedly that the measuring instrument is an inseparable part of the entire system that is described in quantum mechanical terms. He insists that we cannot separate the description of the atomic system from a description of the whole experimental situation, including both the atoms and the apparatus.

Many people do not like this way of talking about physics. My only point here is that there is nothing in this way of speaking that prevents us from doing quantum cosmology. After all, we are in the universe, we are ourselves made of atoms, and we do make observations and describe their results to each other in classical terms. That all these things are true are no more and no less mysterious whether the quantum state is a description of our observations made of the spin of an atom or of the fluctuations in the cosmic black body radiation.

Second objection: *It is inconsistent with the idea that the quantum state describes the whole universe, including us, to postulate that the result of a measurement that we make is one of the eigenvalues of the measured observable, because that is to employ a classical description, while the whole universe is described by a quantum state.* To say so is, in my opinion, again to misunderstand Bohr and von Neumann and, again, to attempt to combine their way of speaking about physics with some postulate about the reality of the quantum state. To postulate that the result of a measurement is an eigenvalue is to assume that the results of measurements may be *described* using quantities from the language of classical physics. The theory does not, and need not, explain to us why that is the case; that we get definite values for the results of experiments we do is taken as a primitive fact upon which we base the interpretation of the theory.

Furthermore, to assume that the results of measurements are *described* in terms of the language of classical physics is not at all the same thing as to make the (obviously false) claim that the dynamics that governs the physics of either the measuring instruments or ourselves is classical.

Third objection: *The fact that we are in the universe might lead to some problem in quantum cosmology because a measurement of a quantum state would involve a measurement of our own state.* There are two replies to this.

First, there is an interpretation of quantum mechanics, suggested by von Neumann[vN] and developed to its logical conclusion by Wigner[EW], that says that all we ever actually do is make observations on our own state. Wigner claims that there is something special about consciousness which is that we can experience only definite things, and not superpositions. This is then taken to be the explanation for why we observe the results of experiments to give definite values. I do not personally believe this point of view, but the fact that it is a logically possible interpretation of quantum mechanics means that there can be no logical problem with including ourselves (and all our measuring instruments and cats, if not friends) in the description of the quantum state.

Second, we can avoid this problem, at least temporarily, if we acknowledge that all measurements we make in quantum cosmology are incomplete measurements. In reality we never determine very much information about the quantum state of the universe when we make a measurement, however we interpret it. We certainly learn very little about our own state when we make a measurement of the gravitational field of the sort I described in section 2.

Thus, as long as we refrain from actually describing experiments in which we make measurements on our own brains, we need not commit ourselves to any claims about the results of making observations on ourselves. Again, the situation here is exactly the same in quantum cosmology as it is in ordinary quantum mechanics-no worse and no better. If there is a possible problem with making observations on ourselves in quantum cosmology, it must occur in ordinary quantum mechanics. And it must be faced there as well, as it cannot matter for the resolution of such a problem whether, besides our brain, Andromeda or the Virgo cluster is also described by the quantum state.

Let me close this section with one comment. Given the measurement postulate above, and the results of the last two sections, we can conclude that in fact the areas of surfaces are quantized in quantum gravity. For, without integrating the evolution equations, we know that $\hat{\mathcal{A}}^I(\tau = 0)_{phys} = \hat{\mathcal{A}}^I(\tau = 0)_{diffeo}$ and the latter operator, from the results of [CR93, LS93] has a discrete spectrum. Thus, quantum gravity makes a physical prediction. Note, further that this result is independent of the form of the Hamiltonian constraint and hence of the dynamics and the matter content of the theory.

5. The recovery of conventional quantum field theory

The measurement theory given in the previous section has not required any notion of classical or semiclassical states. One need only assume that it is possible to prepare the fields that describe the spatial and temporal quantum reference frame appropriately so that subsequent measurements are meaningful. One does not need to assume that the gravitational field is in any particular state to do this. Of course, there may be preparations that require some restriction on the state of the gravitational degrees of freedom, but such an assumption is not required in principle. Further, the examples discussed in the last sections show that there are some kinds of physical reference frames whose preparation requires absolutely no restrictions on the gravitational field.

Having said this, we may investigate what happens to the dynamics and the measurement theory if we add the condition that the state is semiclassical in the gravitational degrees of freedom. I will show in this section that by making the assumption that the gravitational field is in a semiclassical state we can recover quantum field theory for the matter fields on a fixed spacetime background. Thus, quantum cosmology, whose dynamics is contained in the quantum constraint equations, and whose interpretation was described in the previous section, does have a limit which reproduces conventional quantum field theory.

Here I will only sketch a version of the demonstration, as my main motive here is to bring out an interesting point regarding a possible role for the zero point energy in the transition from quantum gravity to ordinary quantum field theory. For simplicity, I will also drop in this section the assumption that the state is diffeomorphism invariant, as this will allow me to make use of published results about the semiclassical limit of quantum gravity in the loop representation. However, I will continue to treat the Hamiltonian constraint in the gauge fixed formalism of sections 3.2-3.4

I will make use of the results described in [ARS92] in which it was shown that, given a fixed three metric q_0^{ab}, whose curvatures are small in Planck units, we can construct, in the loop representation, a nonperturbative quantum state of the gravitational field that approximates that classical metric up to terms that are small in Planck units.

Such a state can be described as follows[15]. Given the volume element $\sqrt{q_0}$, let me distribute points randomly on Σ with a density $1/l^3_{Planck} \times (2/\pi)^{3/2}$. Let me draw a circle around each point with a radius given by $\sqrt{\frac{\pi}{2}} l_{Planck}$ with an orientation in space that is random, given the metric q_0^{ab}. As the curvature is negligible over each of these circles, this is well defined. Let me call the collection of these circles $\Delta = \{\Delta_I\}$, where I labels them.

Let me then define the *weave state associated to* q_0^{ab} as the *characteristic state* of the set of loops Δ. This is denoted χ_Δ and, for a non-selfintersecting loop Δ, it is defined so that $\chi_\Delta[\alpha]$ is an eigenstate of the area operator $\mathcal{A}[\mathcal{S}]$ which measures the area of the arbitrary surface with eigenvalue $l^2_{Planck}/2$ times the number of times the loop Δ intersects the surface $\mathcal{S}$. The result is that $\chi_\Delta[\alpha]$ is equal to one if α is equivalent to Δ under the usual rules of equivalence of loops in the loop representation and is equal to zero for most other loops α (including all other distinct non self-intersecting loops.)

I will also assume that the weave is chosen to have no intersections, in which case

$$\lim_{\epsilon \longrightarrow 0} \mathcal{C}^\epsilon_{grav.}(x) \chi_\Delta[\alpha] = 0 \tag{63}$$

This will simplify our discussion.

I would now like to make the ansatz that the state is of the form

$$\Psi[\gamma, \phi, \tau] = \chi_\Delta[\gamma] \Phi[\gamma, \phi, \tau]. \tag{64}$$

[15] More details of this construction are given in [ARS92]. See also [AA92, CR91, LS91].

Note that, as I have dropped for the moment the requirement of diffeomorphism invariance, I have also dropped the dependence on the spatial reference frame field $\mathcal{S}$.

Now, let me assume, as an example, that the matter consists of one scalar field, called $\phi(x)$, with conjugate momenta $\pi(x)$, whose contribution to the regulated Hamiltonian constraint is,

$$\begin{aligned}\hat{\mathcal{C}}^{\epsilon}_{\phi}(x) &= -\frac{1}{2}\int d^3y\int d^3z f^{\epsilon}(x,y)f^{\epsilon}(x,z) \\ &\quad\times\left[\pi(y)\pi(z)+\hat{T}^{ab}(y,z)\partial_a\phi(y)\partial_b\phi(z)\right]. \end{aligned}\tag{65}$$

Let us focus on the spatial derivative term. Let us assume that the dependence of $\Phi[\gamma,\phi,\tau]$ on the gravitational field loops γ can be neglected. (This is exactly to neglect the back reaction and the coupling of gravitons to the matter field.) Let me assume also that the support of the state on configurations on which the scalar field is not slowly varying on the Planck scale (relative to q_0^{ab}) may be neglected. Then it is not hard to show, following the methods of [ARS92, LS91] that as long as the scalar fields are slowly varying on the Planck scale,

$$\begin{aligned} &\int d^3y\int d^3z f^{\epsilon}(x,y)f^{\epsilon}(x,z)\hat{T}^{ab}(y,z)\partial_a\phi(y)\partial_b\phi(z)\Psi[\gamma,\phi,\tau] \\ =\; &\left(\int d^3y\int d^3z f^{\epsilon}(x,y)f^{\epsilon}(x,z)\hat{T}^{ab}(y,z)\chi_{\Delta}[\gamma]\right)\partial_a\phi(y)\partial_b\phi(z)\Phi[\gamma,\phi,\tau] \\ =\; &\sum_I\sum_J\int ds\int dt f^{\epsilon}(x,\Delta_I(s))f^{\epsilon}(x,\Delta_J(t))\dot{\Delta}^a_I(s)\dot{\Delta}^b_I(t) \\ &\times\left(\sum_{\text{routings}}\chi_{\Delta}[\gamma\circ\gamma_{x,y}]\right)\partial_a\phi(\Delta_I(s))\partial_b\phi(\Delta_J(t))\Phi[\gamma,\phi,\tau] \\ =\; &det(q_0)q_0^{ab}(x)\partial_a\phi(x)\partial_b\phi(x)\chi_{\Delta}[\gamma]\Phi[\gamma,\phi,\tau]+O(l^2_{Planck}\partial_a\phi) \end{aligned}\tag{66}$$

Putting these results together, we have shown that if we make an ansatz on a state of the form of (64) then, neglecting the dependence of Φ on γ, the regulated Hamiltonian constraint (36) is equivalent to

$$\imath\frac{d\Phi[\gamma,\phi,\tau]}{d\tau}=\sqrt{2\mu}\int d^3x\sqrt{\left[\frac{1}{2}\hat{\pi}^2(x)+\frac{1}{2}det(q_0)q_0^{ab}(x)\partial_a\phi(x)\partial_b\phi(x)\right]}\Phi[\gamma,\phi,\tau]+... \tag{67}$$

This does not yet look like the functional Schroedinger equation for the scalar field. However, we may recall that formally the expression inside the square root is divergent. However, it may not be actually divergent because in computing (66) we have assumed that the scalar field is slowly varying on the scale of the weave. If we investigate the action of (65) on states which have support on $\phi(x)$ that are fluctuating on the Planck scale, we can see that in the limit that the regulator is removed the effect of the T^{ab}'s is to insure that the the terms in $(\partial_a\phi)^2$ only act at those points which are on the lines of the weave. That is, in the limit of small distances we have a description of a scalar field propagating on a one dimensional subspace of Σ picked out by the weave. That is, on scales much smaller than the Planck scale the scalar field is propagating as a $1+1$ dimensional scalar field.

The result must be to cut off the divergence in the zero point energy coming formally from the scalar field Hamiltonian. The effect of this must be the following: If we decompose the scalar field operators into creation and annihilation operators defined with respect to the background metric q_0^{ab} that the weave corresponds to, then the divergent term in the zero point energy must cut off at a scale of M_{Planck}. As such, we will have, if we restrict attention to the action of (67) on states that are slowly varying on the Planck scale

$$\begin{aligned} \frac{1}{2}\hat{\pi}^2(x) \quad + \quad & \frac{1}{2}det(q_0)q_0^{ab}(x)\partial_a\phi(x)\partial_b\phi(x) = aM_{Planck}^4 \\ & + : \frac{1}{2}\hat{\pi}^2(x) + \frac{1}{2}det(q_0)q_0^{ab}(x)\partial_a\phi(x)\partial_b\phi(x) : \end{aligned} \tag{68}$$

where : ... : means normal ordered with respect to the background metric and a is an unknown constant that depends on the short distance structure of the weave.

The reduced Hamiltonian constraint now becomes,

$$\begin{aligned} \imath\frac{d\Phi[\gamma,\phi,\tau]}{d\tau} &= \sqrt{2a\mu}M_{Planck}^2 \\ &\times \int d^3x\sqrt{1+\frac{1}{aM_{Planck}^4} : \left(\frac{1}{2}\hat{\pi}^2(x) + \frac{1}{2}det(q_0)q_0^{ab}(x)\partial_a\phi(x)\partial_b\phi(x)\right) :}\Phi[\gamma,\phi,\tau] + ... \\ &= \sqrt{2a\mu}M_{Planck}^2 V\Phi[\gamma,\phi,\tau] \\ &+ \frac{\sqrt{\mu}}{M_{Planck}^2\sqrt{2a}}\int d^3x\sqrt{q_0} : \left(\frac{1}{2}\hat{\pi}^2(x) + \frac{1}{2}det(q_0)q_0^{ab}(x)\partial_a\phi(x)\partial_b\phi(x)\right) : \Phi[\gamma,\phi,\tau] \\ &+ ... \end{aligned} \tag{69}$$

where $V = \int \sqrt{q_0}$ is the volume of space.

Thus, only after taking into account the very large zero point energy do we recover conventional quantum theory for low energy physics.

Before closing this section, I would like to make three comments on this result.

1) Note that the theory we have recovered is Poincare invariant, even if the starting point is not! We may note that the weave state χ_Δ is *not* expected to be the vacuum state of quantum gravity because it is a state in which the spatial metric is sharply defined. What we need to describe the vacuum is a Lorentz invariant state which which is some kind of minimal uncertainty wave packet in which the three metric and its conjugate momenta are equally uncertain. A state that has these properties, at least at large wavelengths, can be constructed by dressing χ_Δ with a Gaussian distribution of large loops that correspond to a Gaussian distribution of virtual gravitons [ASR91]. It is interesting to note that at the level when we neglect the back-reaction and the coupling to gravitons, the Poincare invariant matter quantum field theory is nevertheless recovered by using the weave state χ_Δ as the background. However, before incorporating quantum back-reaction and the coupling to gravitons we must replace χ_Δ in (64) with a good approximation to the vacuum state, such as is described in [IR93].

2) Can we add a mass term and self-interaction terms for the scalar field theory? The answer is yes, but to do so we must modify the weave construction in order to add intersections. The reason is that the scalar mass and self interaction is described by the term $\hat{q}V(\hat{\phi})$, where $\hat{q}$ is the operator corresponding to the determinant of the metric. Using results about the volume operator in [LS91], it is easy to see that if we modify the weave construction in order to add intersections, then the effect of this term, after regularization, is to modify $\int d^3xN(x)\mathcal{C}_{matter}\Psi$ by the addition of the term

$$l^3_{Planck}\sum_i a(i)N(x_i)V(\phi(x_i))\Phi \tag{70}$$

where the sum is over all intersections involving three or more lines, x_i is the intersection point and $a(i)$ are dimensionless numbers of order one that characterize each intersection. Assuming that there are on the order of $a(i)^{-1}$ intersection points per Planck volume, measured with respect to the volume element q_0 (which is consistent with the weave construction described above as that is the approximate number of loops) we arrive at an addition to (69) of the form of $N(x)q(x)V(\phi(x))\Phi$.

Note that once we add intersections that produce volume it is no longer true that the gravitational part of the Hamiltonian constraint is solved by χ_Δ. This is because there is now a term in the back-reaction of the quantum matter field on the metric coming from the local potential energy of the scalar field. This is telling us that we now cannot neglect the back-reaction of the quantum fields on the background metric to construct solutions of the Hamiltonian constraint.

3) Finally, let me note that the measurement theory of the semiclassical state (64) is already defined because we have a measurement for the full nonperturbative theory. We therefore do not have to supplement the derivation of the equations of quantum mechanics from solutions to the quantum constraint equations of quantum cosmology with the *ab initio* postulation of the standard rules of interpretation of quantum field theory. This is always a suspicious procedure as those rules rely on the background metric that is only a property of a particular state of the form of (64); we cannot then choose inner products or other aspects of the interpretative machinery to fit a particular state.

In this case, since we already have an inner product and a set of rules of interpretation defined for the full theory, what needs to be done is to verify that the usual quantum field theory inner product is recovered from the full physical inner product defined by (46) in the case that the state is of the form of (64) . We have seen in section 3 that this will be the case to the extent to which $\mathcal{W}$ defined by (37) is hermitian. We see that in the approximation that leads to (69), the contribution to $\mathcal{W}$ from the matter fields is hermitian a long as the diffeomorphism inner product implies that the operators for ϕ and π are also hermitian. In this case, then, the usual inner product of quantum field theory must be recovered.

6. Singularities in quantum cosmology

Having established a physical interpretation for quantum cosmology and shown that it leads to the recovery of conventional quantum field theory in appropriate circumstances, we now have tools with which to address what is perhaps the key problem that any quantum theory of quantum gravity must solve, which is what happens to black holes and singularities in the quantum theory.

The main question that must be answered is to what extent the apparent loss of information seen in the semiclassical description of black hole evaporation survives, or is resolved, in the full quantum theory. The key point that must be appreciated to investigate this problem from the fully quantum mechanical point of view is that the problem of loss of information, or of quantum coherence, is a problem about time because the question cannot be asked without assuming that there is a meaningful notion of time with respect to which we can say information or coherence is being lost. If we take an operational approach to the meaning of time in the full quantum theory, along the lines that have been developed here, then the loss of information or coherence, if it exists at the level of the full quantum theory, must show up as a limitation on the possibility of completely specifying the quantum state of the system by measuring the physical observables, $\mathcal{O}(\tau)_{phys}$ for sufficiently late τ.

What I would like to do in this section is to describe how singularities, if they occur in the full quantum theory, will show up in the action of these physical time dependent observables. The result will be the formulation of two conjectures about how singularities may show up in the quantum theory, which I will call the *quantum singularity conjecture* and the *quantum cosmic censorship conjecture*.

While I will not try to prove these conjectures here, I also will argue that there is no evidence that they may not be true. It is possible that quantum effects completely eliminate the singularities of the classical theory as well as the consequent losses of information and coherence in the semiclassical theory. But, it seems to me, it is at least equally possible that the quantum theory does not eliminate the singularities. Rather, given the formulation of the theory along the lines described in this paper, the occurrence of singularities and loss of information, as formulated in these two conjectures, seems to be compatible with both the dynamics and interpretational framework of quantum cosmology.

The loss of information and coherence in the semiclassical theory implies a breakdown in unitarity in any process that describes a black hole forming and completely evaporating. From a naive point of view, this would seem to indicate a breakdown in one of the fundamental principles of quantum mechanics. Hence many discussions about this problem seem to assume that if there is to be a good quantum theory of gravity it must resolve this problem in such a way that unitarity is restored in the full quantum theory. However, the results of the last several sections show that unitarity is not one of the basic principles of quantum cosmology. It is not because unitarity depends on a notion of time that apparently cannot be realized in either classical or quantum cosmology.

To put it most simply, if the concept of time is no longer absolute, but depends for its properties on certain contingent facts about the universe, principally the existence of degrees of freedom that behave as if there is a universal and absolute Newtonian notion of time, then the same must be true for those structures and principles that, in the usual formulation of quantum mechanics, are tied to the absolute background time of the Schroedinger equation. Chief among these are the notions of unitarity evolution and conservation of probability.

As I have argued in detail elsewhere[LS91, LS92c], conventional quantum mechanics, no less than Newtonian mechanics, relies for its interpretation on the assumption of an absolute background time. When we speak of conservation of probability, or unitary evolution in quantum mechanics, we do not have to ask whether something might happen to the clocks that measure time that could make difficulties for our understanding of the operational meaning of these concepts. A single clock could break, but the t in Schroedinger's equation refers to no particular clock but instead to an absolute time that is presumed to exist independently of the both the physical system described the quantum state and of the physical properties of any particular clock. However, neither in classical nor in quantum cosmology does there seem to be available such an absolute notion of time. In any case, if it exists we have not found it. If we then proceed by using an operational notion of time as I have done here then, because that clock must, by diffeomorphism invariance, be a dynamical part of the system under study, we must confront the question: what happens to the notion of time and to all that depends on it if something happens to the clock whose motion is taken as the operational basis of time. Of course, in the theory, as in real life, in most circumstances in which a clock may fail we can imagine constructing a better one. The problem we really have to face as theorists is not the engineering problem of modeling the best possible clock. The problem we have to face is what the implications are for the theory if there are physical effects that can render useless any conceivable clock.

Of course, in classical general relativity there is such an effect; it is called gravitational collapse. Thus, to put the point in the simplest possible way, in quantum cosmology a breakdown of unitarity need not indicate a breakdown in the theory. Rather, it may only indicate a breakdown in the physical conditions that make it possible to speak meaningfully of unitary evolution. This will be the case if there are quantum states in which some of the physical clocks and some of the components of the physical reference frame that make observations in quantum cosmology meaningful cease to exist after certain physical times. This will not prevent us from describing the further evolution of the system in terms of operators whose meaning is tied to the physical clocks that happen to survive the gravitational collapse. But it will prevent us from describing that evolution in terms of the unitary evolution of the initial quantum state.

In this section I proceed in two steps. First, before we describe how singularities may show up in the operator algebra of the quantum theory, we should see how they manifest themselves in terms of the observable algebra of the classical theory. This then provides the basis for the statements of the quantum singularity conjecture and the quantum cosmic censorship conjecture.

6.1. Singularities in the classical observable algebra

In section 3 we found an evolution equations for classical observables in the gauge defined by (19) and (21). We then used this to define both gauge fixed and Dirac observables that correspond to making measurements on the surface defined by $T(x) = \tau$, given that the clocks are synchronized by setting $T(x) = 0$ on the initial surface. In this section I would like to discuss what effects the singularities of classical general relativity have on these observables[16].

Let us begin with a simple point, which is the following: *Given that the matter fields, including the time fields, satisfy the positive energy condition required by the singularity theorems, for any τ_0 there are regions, $\mathcal{R}(\tau_0)$ of the phase space $\Gamma = \{A, E, \phi, ...)$ such that the future evolution of any data in this region becomes singular before the physical time τ_0 (defined by the gauge conditions (19) and (21)). This means that the evolutions of the data in $\mathcal{R}(\tau_0)$ do not have complete $T(x) = \tau$ surfaces.* This happens because, roughly speaking, some of the clocks that define the $T(x)$ =constant surfaces have encountered spacetime singularities on surfaces with $T(x) < \tau_0$.

Now, let us consider what I will call "quasi-local" observables $\mathcal{O}(\tau)_{phys}$, such as $\mathcal{A}^I(\tau)_{phys}$ and $T^I(\tau)_{phys}$, which are associated with more or less local regions of the initial data surface. We may expect that for such observables the following will be the case: For each such $\mathcal{O}(\tau)_{phys}$ and for each τ_0 there will be regions on the phase space Γ on which $\mathcal{O}(\tau_0)_{phys}$ is not defined because, for data in that region, the local region measured by that observable (picked out by the spatial reference frame) encountered a curvature singularity at some time $\tau_{sing} < \tau_0$.

Thus, it is clear that the existence of spacetime singularities does limit the operational notion of time tied to a field of clocks. The limitation is that, if we define the evolution of physical quantities in terms of the physical observables $\mathcal{O}(\tau)_{phys}$, only the $\tau = 0$ observables that measure the properties of the initial data surface can be said to give good coordinates for the full space of solutions. If we want a complete description of the full space of solutions (defined as the evolutions of non-singular initial data) we cannot get complete information from the evolving constants of motion for any nonzero τ. What we cannot get is complete information about those solutions for which by τ some of the clocks have already fallen into singularities.

As far as the classical theory is concerned, this limitation is necessary and entirely unproblematic. The observables become ill defined because we cannot ask any question about what is seen by observers after they have ceased to exist. As general relativity is a local theory, if we choose our observables appropriately, we can still have complete information about all measurements that can be made at a time τ by local observers who have not yet fallen into a singularity.

I now turn to a consideration of the implications of this for the quantum theory.

6.2. Singularities in quantum observables

We have seen how the existence of singularities in classical theory is expressed in terms of the classical observables $\mathcal{O}(\tau)_{phys}$. There are now two question that must be asked: First, in principle can singularities show up in the same way in the physical operators of a quantum cosmological theory? Second, do they actually occur in the physical operator algebra of a realistic theory of quantum gravity coupled to matter fields?

[16] As in section 4, I will in this section use the notation *phys* to refer either to the gauge fixed or the Dirac formalisms.

The answer to the first question is yes, as has been shown in two model quantum cosmologies. These are a finite dimensional example, the Bianchi I quantum cosmology[ATU] [17] and an infinite dimensional field theoretic example, the one polarization Gowdy quantum cosmology[VH87]. These are both exactly solvable systems; the first has a physical Hilbert space isomorphic to the state space of a free relativistic particle in $2+1$ dimensional Minkowski spacetime while the Hilbert space of the second is isomorphic to that of a free scalar field theory in $1+1$ dimensions. However, in spite of the existence of these isomorphisms to manifestly non-singular physical systems, they are each singular theories when considered in terms of the operators that represent observables of the corresponding cosmological models. In both cases there is a global notion of time, which is the volume of the universe, V in a homogeneous slicing. One can then construct a physical observable, called $C^2(V)$ which is defined as

$$C^2(V) \equiv \int_{\Sigma(V)} \sqrt{q} g^{\mu\nu} g^{\alpha\beta} C^{\rho}_{\sigma\mu\alpha} C^{\sigma}_{\rho\nu\beta} \tag{71}$$

where $\Sigma(V)$ is the three surface defined in the slicing by the condition that the spatial volume is V and $C^{\sigma}_{\rho\nu\beta}$ is the Weyl curvature. There is also in each case, a V-time dependent Hamiltonian that governs the evolution of operators such as $C^2(V)$ through Heisenberg equations of motion.

Now, it is well known that in each of these models the cosmological singularity of the classical theory shows up in the fact that $\lim_{V\rightarrow 0} C^2(V)$ is infinite. What is, perhaps, surprising is that the quantum theory is equally singular, in that $\lim_{V\rightarrow 0} \hat{C}^2(V)$ diverges. The exact meaning of this is slightly different in the finite dimensional and the quantum field theoretic examples. In the Bianchi I case, it has been shown by Ashtekar, Tate and Uggla[ATU] that for any two normalizable states $|\Psi>$ and $|\Phi>$ in the physical Hilbert space

$$\lim_{V\rightarrow 0} < \Psi|\hat{C}^2(V)|\Phi>_{physical} = \infty \tag{72}$$

In the one polarization Gowdy model, Husain [VH87] has shown that (given a physically reasonable ordering for $\hat{C}^2(V)$) there is a unique state $|0>$ such that (72) holds for all $|\Psi>$ and $|\Phi>$ which are not equal to $|0>$. Furthermore, for all V, $<0|\hat{C}^2(V)|0> = 0$ so the state $|0>$ represents the vacuum in which no degrees of freedom of the gravitational degrees of freedom are ever excited. As the Gowdy cosmology contains only gravitational radiation, this corresponds to the one point of the classical phase space which is just flat spacetime. For any other states, the quantum cosmology is singular in the sense that the matrix elements of the Weyl curvature squared diverge at the same physical time that the classical singularities occur.

Thus, we see from these examples that there is no principle that prevents spacetime singularities from showing up in quantum cosmological models and that they manifest themselves in the physical quantum operator algebra in the same way they do in the classical observable algebra. It is therefore a dynamical question, rather than a question of principle, whether or not singularities can occur in the full quantum theory. While it is, of course, possible that the singularities are eliminated in every consistent quantum theory of gravity, I think it

[17] Similar phenomena occur for other Bianchi models[ATU].

must be admitted that at present there is little evidence that this is the case. The evidence presently available about the elimination of singularities is the following: a) Cosmological singularities are not eliminated in the semiclassical approximation of quantum cosmology [HH]. b) There are exact solutions of string theory which are singular [GH92]. c) In $1+1$ models of quantum gravity, singularities are sometimes, but not always, eliminated, at the semiclassical level[EW91, GH92, HS92, JP92].

Furthermore, there does not seem to be any reason why quantum cosmology requires the removal of the singularities. Both the mathematical structure and the physical interpretation of the quantum theory are, just like those of the classical theory, robust enough to survive the occurrence of singularities.

Given this situation, it is perhaps reasonable to ask whether and how singularities may appear in full quantum gravity. As a step towards answering this question, we may postulate the following conjecture:

Quantum singularity conjecture *There exist, in the Hilbert space $\mathcal{H}_{phys}$ of quantum gravity, normalizable states $|\Psi>$ such that:*

a) the expectation values $<\Psi|\hat{\mathcal{O}}_I(0)_{phys}|\Psi>_{phys}$ are finite for all I, so that at the initial time $\tau=0$, all physical observables are finite.

b) There is a subset of the physical operators, $\hat{\mathcal{O}}_I(\tau)_{phys}$, whose expectation values in the state $|\Psi>$ develops singularities under evolution in the physical time τ. That is, for each such $|\Psi>$ and for each $\hat{\mathcal{O}}_I(\tau)_{phys}$ in this subset there is a finite time τ_{sing} such that

$$\lim_{\tau\rightarrow\tau_{sing}} <\Psi|\hat{\mathcal{O}}^I(\tau)_{phys}|\Psi>_{phys}=\infty \tag{73}$$

This means that if we want to predict what observers in the universe described by the state $|\Psi>$ will see at some time τ_0, they may measure only the $\hat{\mathcal{O}}_I(\tau_0)_{phys}$ which do not go singular in this sense by the time τ_0. This means that they may be able to recover less information about the state $|\Psi>$ by making measurements at that time than was available to observers at the time $\tau=0$. Thus, the occurrence of singularities in the solutions to the operator evolution equations (43) (or (51)) means that real loss of information happens in the full quantum theory.

The possibility of this happening can be captured by a conjecture that I will call the *quantum cosmic censorship conjecture.* The name is motivated by analogy to the classical conjecture: if there is censorship then there is missing information. This is the content of the following:

Quantum cosmic censorship conjecture: a) *There exists states $|\Psi>$ in $\mathcal{H}_{phys}$ which are singular for at least one observable $\mathcal{O}_{sing}(\tau)_{phys}$ at some time τ_{sing}, but for which there are a countably infinite number of other observables $\mathcal{O}'_I(\tau)$ such that $<\Psi|\mathcal{O}_I(\tau)_{phys}|\Psi>$ are well defined and are finite for some open interval of times $\tau>\tau_{sing}$.*

b) *Let* $|\Psi>$ *be a state which satisfies these conditions. Then for every* τ *in this open interval there is a proper density matrix* $\rho_\Psi(\tau)$ *such that, for every* $\hat{\mathcal{O}}_I(\tau)$*, for which* $<\Psi|\hat{\mathcal{O}}_I(\tau)|\Psi>$ *is finite, then*

$$<\Psi|\hat{\mathcal{O}}_I(\tau)|\Psi>_{phys}= Tr\rho_\Psi(\tau)\hat{\mathcal{O}}_I(\tau). \quad (74)$$

Here the trace is to be defined with respect to the physical inner product a proper density matrix is one that corresponds to no pure state.

The first part of the conjecture means that there are states which describe what we might want to call black holes in the sense that while some observables become singular at some time τ_{sing} there are other observables which remain nonsingular for later times. The second part means that there exists a density matrix that contains all the information about the quantum state that is relevant for physical times τ after the time of the first occurrence of singularity. Because a density matrix contains all the information that could be gotten by measuring the pure state, we may say that loss of information has occurred.

Finally, we may note that for any state $|\Psi>$ there may be a finite time τ_{final} such that, for every observable $\mathcal{O}_I(\tau)_{phys}$, $<\Psi|\mathcal{O}_I(\tau)_{phys}|\Psi>_{phys}$ is undefined, divergent or zero for every $\tau>\tau_{final}$. This would correspond to a quantum mechanical version of a final singularity.

Suppose these conjectures can be proven for quantum general relativity, or some other quantum theory of gravity. Would this mean that the theory would be inadequate for a description of nature? While someone may want to argue that it may be preferable to have a quantum theory of gravity without singularities, I do not think an argument can be made that such a theory must be either incomplete, inconsistent or in disagreement with anything we know about nature. What self-consistency and consistency with observation require of a quantum theory of cosmology is much less. The following may be taken to be a statement of the minimum that we may require of a quantum theory of cosmology:

Postulate of adequacy. A quantum theory of cosmology, constructed within the framework described in this paper, may be called *adequate* if,

a) *For every* $\tau>0$ *there exists a physical state* $|\Psi>\in\mathcal{H}_{phys}$ *and a countable set of operators* $\hat{\mathcal{O}}_I(\tau)$ *such that the* $<\Psi|\hat{\mathcal{O}}_I(\tau)_{phys}|\Psi>_{phys}$ *are finite.*

b) *The theory has a flat limit, which is quantum field theory on Minkowski spacetime. This means that there exists physical states and operators whose expectation values are equal to those of quantum field theory on Minkowski spacetime for large regions of space and time, up to errors which are small in Planck units.*

c) *The theory has a classical limit, which is general relativity coupled to some matter fields. This means that there exists physical states and operators whose physical expectation values are equal to the values of the corresponding classical observables evaluated in a classical solution to general relativity, up to terms that are small in Planck units.*

If a theory satisfies these conditions, we would have a great deal of trouble saying it was not a satisfactory quantum theory of gravity. Thus, just like in the classical theory, the presence of singularities and loss of information cannot in principle prevent a quantum theory of cosmology from providing a meaningful and adequate description of nature. Whether there is an adequate quantum theory of cosmology that eliminates singularities and preserves information is a dynamical question, and whether that theory, rather than another adequate theory for which the quantum singularity and quantum cosmic censorship conjectures hold, is the correct description of nature is, in the end, an empirical question.

7. Conclusions

The purpose of this paper has been to explore the implications of taking a completely pragmatic approach to the problem of time in quantum cosmology. The main conclusion of the developments described here is that such an approach may be possible at the non-perturbative level. This may allow the theory to address problems such as the effect of quantum effects on singularities which most likely require a nonperturbative treatment, while remaining within the framework of a coherent interpretation.

I would like to close this paper by discussing two questions. First, are there ways in which the ideas described here may be tested? Second, is it possible that there is a more fundamental solution to the problem of time in quantum gravity which avoids the obvious limitations of this pragmatic approach?

7.1. Suggestions for future work

The proposals and conjectures described here are only meaningful to the extent that they can be realized in the context of a full quantization of general relativity or some other quantum theory of gravity. In order to do this, the key technical problem that must be resolved is, as we saw in section 3, the construction of the operator $\mathcal{W}$. Given that we know rather a lot about both the kernel and the action of the Hamiltonian constraint, I believe that this is a solvable problem.

Beyond this, it would be very interesting to test these ideas and conjectures in the context of certain model systems. Among those that could be interesting are 1) The Bianchi IX model 2) The full two polarization Gowdy models 3) Models of spherically symmetric general relativity coupled to matter[18] 4) Other $1+1$ dimensional models of quantum gravity coupled to matter such as the dilaton theories that have recently received some attention [CGHS, GH92, HS92, B92, ST92, SWH92]. 5) the chiral $G \to 0$ limit of the theory[LS92d] and finally, 6) $2+1$ general relativity coupled to matter[EW88, AHRSS]. Each of these are systems that have not yet been solved, and in which the difficulty of finding the physical observables has, as in the full theory, blocked progress.

[18] For the complete quantum theory without matter, see [TT].

7.2. Is there an alternative framework for quantum cosmology not based on such an operational notion of time?

We are now, if the above is correct, faced with the following situation. Taking an operational approach to the meaning of time we have been able to provide a complete physical interpretation for quantum cosmology that reduces to quantum field theory in a suitable limit. However, the quantum field theory that is reproduced may turn out to be unitary only in the approximation in which we can neglect the possibility that some of the clocks that define operationally surfaces of simultaneity become engulfed in black hole singularities. This need not be disturbing; it says that we cannot count on probability conservation in time if the notion of time we are thinking of is based on the existence of a certain field of dynamical clocks and there is finite probability that these clocks themselves cease to exist. However, if there is no other notion of time with respect to which probability conservation can be maintained, so that this is the best that can be done, it is still a bit disturbing.

We seem at this juncture to have two choices. It may indeed be that we cannot do better than this, so that we must accept that the Hilbert space structure that forms the basis of our interpretation of quantum cosmology is tied to the existence of certain physical frames of reference and that, as the existence of the conditions that define these frames of reference is contingent, we cannot ascribe any further meaning to unitarity. If this is the case then we have to accept a further "relativization" of the laws of physics, in which different Hilbert structures, with different inner products, are associated to observations made by different observers. This means, roughly, that not only are the actualities (to use a distinction advocated by Shimony[Sh]) in quantum mechanics dependent on the physical conditions of the observer, so are the potentialities. This point of view has been advocated by Finkelstein[F] and developed mathematically in a very interesting recent paper of Crane[LC92].

On the other hand, it is possible to imagine that there is some meaning to what the possibilities are for actualization that is independent of the conditions of the observer. If such a level of the theory existed, it could be used to deduce the relationships between what could be, and what is, seen by different observers in the same universe.

Barbour has recently made a proposal about the role of time in quantum cosmology, which I think can be understood along these lines[JBB92]. I would now like to sketch it, as it may serve as a prototype for all such proposals in which the probability interpretation of quantum cosmology is not relativized so as to make the inner produce dependent on the conditions of the observer.

Barbour's proposal is at once a new point of view about time and a new proposal for an interpretation of quantum cosmology. He posits that time actually does not exist, so that our impressions of the existence and passage of time are illusions caused by certain properties of the classical limit of quantum cosmology. More precisely, he proposes that an interpretation can be given entirely in the context of the diffeomorphism invariant theory.

Barbour's fundamental postulate, to which he gives the colorful name of the "heap hypothesis", is as follows: *The world consists of a timeless real ensemble of configurations, called "the heap".* The probability for any given spatially diffeomorphism invariant property to occur in "the heap" is governed by a quantum state, $|\Psi >$ which is assumed to satisfy all the constraints of quantum gravity. This probability is considered to be an actual ensemble average. Given a particular diffeomorphism invariant observable $\hat{\mathcal{O}}_I$, the ensemble expectation value in the heap is given by

$$< \Psi|\hat{\mathcal{O}}_I|\Psi >_{complete\ diffeo} . \tag{75}$$

Here the inner product is required to be the spatially diffeomorphism invariant inner product for the whole system, including any clocks that may be around. Thus, this proposal is different than the one made in section 3 in which the inner product proposed in (46) is the spatially diffeomorphism invariant inner product for a specifically reduced system in which the clock has been removed.

There are several comments that must be made about this proposal. First, this is the complete statement of the interpretation of the theory. Quantum cosmology is understood as giving a statistical description of a real ensemble of configurations, or moments. There is no time. The fact that we have an impression of time's passage is entirely to be explained by certain properties of the quantum state of the universe. In particular, Barbour wants to claim that our experience of each complete moment is, so to speak, a world unto itself. It is only because we have memories that we have an impression in this moment that there have been previous moments. It is only because the quantum state of the universe is close to a semiclassical state in which the laws of classical physics approximately hold that the world we experience at this moment gives us the strong impression of causal connections to the other moments.

Second, the probabilities given by (75) are not quantities that are necessarily or directly accessible to observers like ourselves who live inside the universe. Only an observer who is somehow able to look at the whole ensemble is able to directly measure the probabilities given by (75). Of course, we are not in that situation. Barbour must then explain how the probabilities for observations that we make are related to the probability distribution for elements of the heap to have different properties. In order to do this, the key thing that he must do is show how the probabilities defined by the heap ensemble (75) are related to what we measure.

One way in which this may happen is that if one considers only states in which some variables corresponding to a particular clock are semiclassical, then Barbour's inner product (75) may reduce to the inner product defined by (46) in which the clock degrees of freedom have been removed. From Barbour's point of view, the inner product proposed in this paper could only be an approximation to the true ensemble probabilities (75) that holds in the case that the quantum state of the universe is semiclassical in the degrees of freedom of a particular physical clock.

This discussion of Barbour's proposal brings us back to the choice I mentioned above and points up what I think is a paradox that must confront any quantum theory of cosmology. The two possibilities we must, it seems, choose between can be described as follows: a) The inner product and the resulting probabilities are tied in an operational sense to what may be seen by a physical observer inside the universe. In this case, as we have seen here, the notions of unitarity and conservation of probability can only be as good as the clocks carried by a particular observer may be reliable. We are then in danger of the kind of relativization of the interpretation mentioned above. b) The basic statement of the interpretation refers neither to a particular set of observers nor to time as measured by clocks that they carry. In this case the relativization can be avoided, but at the cost that the fundamental quantities of the theory do not in general refer to any observations made by observers living in the universe. In this case the probabilities seen by any observers living in the universe can only approximate the true probabilities for particular semiclassical configurations. Furthermore, if there is a finite probability that any physical clock may in its future encounter a spacetime singularity, however small, then it is difficult to see how a breakdown of unitarity evolution can be avoided, if by evolution we mean anything tied to the readings of a physical clock.

This situation brings us back to the problem of what happens to the information inside of an evaporating black hole. I think that the minimum that can be deduced from the considerations of section 6 is that, at the nonperturbative level, this question cannot be resolved without resolving the dynamical question of what happens to the singularities inside classical black holes. As pointed out a long time ago by Wheeler[JAW], this problem challenges all of our ideas about short distance physics and its relation to cosmology[19]. Unfortunately, it must be admitted that the quantum theory of gravity still has little to say about this problem. What I hope to have shown here is that there may be a language which allows the problem to be addressed by a nonperturbative formulation of the quantum theory. Whether it can be answered by such a formulation remains a problem for the future.

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